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HALL TRIPLE SYSTEMS

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Given a point-line space requirement that every line is of size 3, i.e. there are exactly 3 points on every line, reveals numerous surprising properties. Such point-line spaces are called Steiner Triple Systems. They were introduced in XIX-th century independently by Woolhouse and Steiner. Since then they have gained a wide recognition in the literature thanks to numerous applications in algebra, combinatorics, graph theory and geometry. From an algebraic point of view they are interesting as they are closely related to loops and quasi-groups [1]. Indeed the operation of *the third point on a line* has remarkable algebraic properties. On the other hand we have so many triple systems in geometry: projective, affine and those which emerge by requirement that on each point there is a reflection; the latter called Hall Triple Systems [2].

All we need to distinguish projective, affine and Hall triple systems are pure and elementary geometric properties. These specific triple systems can be characterized by configurations they always contain, or configurations they could never contain [3, 4]. This is a manifestation of how powerful simple geometry could be. Another view of Hall triple systems is through symplectic trilinear forms [1]. To some degree these two notions are different faces of the same phenomenon.

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