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A CONCEPTUAL FRAMEWORK FOR THE GRAPHIC REPRESENTATION OF A STRUCTURAL NODE BASED ON THE GEOMETRY OF THE TETRAHEDRON.

Key words: Structural node, Three-dimensional structures, Rapid prototyping.

Abstract: Taking into account the principle that the triangle is the only polygon that does not deform when a force is applied, we designed a structural node, based on the geometry of the tetrahedron, to generate through the resources and elements of descriptive geometry, spatial structures using CAD applications and rapid prototyping

1. Introduction

Throughout history, the development of spatial structures is linked to technological changes and the process of construction of the moment.

When we talk about spatial structures, we associate them with armors. A truss is a structural grid of straight bars interconnected with nodes forming triangles. The elements form, usually, one or more triangles in a single plane and are arranged so that the external loads are applied to the nodes.

The nodes are defined as the structural part to which are connected the ends of the bars, which together, due to their loads and reactions, form a system of concurrent forces.

From the point of view of engineering and architecture, almost nothing has been worked on the design of the nodes for spatial structures, so the purpose of this research is to design a structural node, based on the geometry of the tetrahedron, to generate spatial structures, considering the law constructive regular spatial structures (Trigonal system), discovered by Max Mengeringhausen (1957) [5] that says: the spatial structures are perfect when combined triangles are formed so that put together, forming octahedrons, tetrahedrons or cubes.

The aims of this project is to investigate, experiment, explore and evaluate a structural node, from the point of view of modeling and the creation of rapid prototyping, to generate spatial structures, as cube, tetrahedron and sphere.

This paper is organized as follows: in section 2 gives a brief explanation of the tetrahedron, in Section 3, we explain the mathematical principles that were used to find the three tetrahedrons. In Section 4, it is illustrates the construction of the structural node from the standpoint of mathematics and design using CAD applications. Section 5 shows the final three-dimensional structure. Section 6 presents the results and finally Section 7, the conclusions. We want to mention that all the figures presented in this paper are original and created by the authors at the Autonomous Metropolitan University, Cuajimalpa in Mexico City.

2. Trace of the tetrahedron

A tetrahedron is a polyhedron formed by four equilateral triangles, and four vertices. It is one of five perfect polyhedral called Platonic solids, and meets the Euler polyhedron theorem, (c + v = a + 2, where "c" is the number of faces, "v" is the number of vertices and "a" is the number of edges) 4 + 4 = 6 + 2.

For the trace of the tetrahedron, it is necessary to place in the horizontal plane of projection, one of its edges parallel to either axis, "x", "y" "z" in order to determine its height in the vertical or lateral view to create the orthographic projection.

We find the height of the tetrahedron in the lateral view because the edge is parallel to the axis "y", and we find the true form and magnitude of the plane and the edges of the tetrahedron, by the method of change of plans [1], but can also be performed by the method of rotation, to find the center of the tetrahedron and the center of each of the sides in each plane of projection, to trace the three tetrahedrons we use in the design of the structural node as explained in the following section.

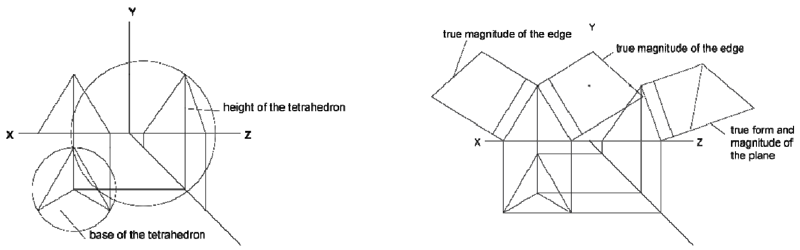


Fig. 1 Tetrahedron one: (a) Trace, (b) True form and magnitude of the plane and edges.

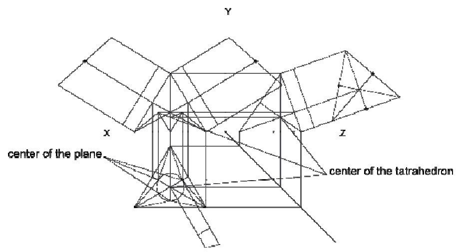


Fig. 2 Center of the tetrahedron one

3. Mathematical principles

The Thales of Miletus theorem says that if three lines, a, b, c, cut two secant lines r, r ', the resulting segments are proportional [4].

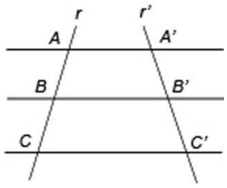


Fig. 3 Miletus´ theorem: Proportional segments.

Then, if we divide the true magnitude of the edge of the tetrahedron, segment AB, in a given ratio in the third part, the length of the line segment is 2/3, and the point "p" divides the segment AB at a ratio of 1:2, which in turn gives us the true magnitude of the edge of the tetrahedron two in a three part ratio, in the lateral plane of projection:

$$r = AP/AB \quad r = ((1/3)/2)/3 \quad r = 1/2$$

If we make this same process in the true magnitude of the edge of the tetrahedron two, we find the true magnitude of the edge of the tetrahedron three, in a ratio of three, in the lateral plane of projection.

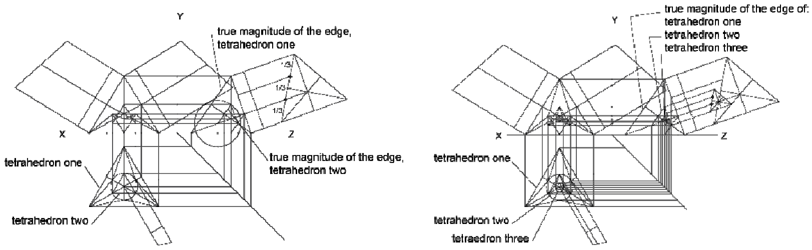


Fig. 4 Tetrahedron two and three: (a) Trace, (b) True form and magnitude of the plane and edges

We can then say, if we divide the true magnitude of the side of the tetrahedron in three and this by three, we found that the rate at which decreases the size of tetrahedron in space is in a ratio of three:

$$d = 259.81/3 = 86.6/3 = 28.87$$

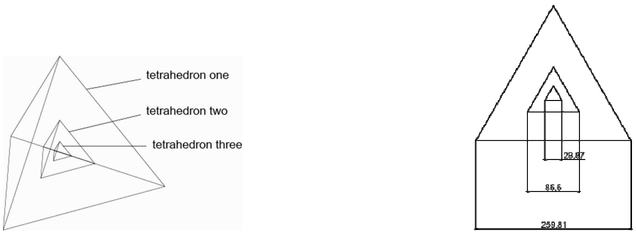


Fig. 5 Tetrahedron one, two and three: (a) Three-dimensional graphical representation, (b) Ratio of three.

4. Construction of the structural node

This section will begin by presenting the render and orthogonal representations in two and three-dimensional “Structural node”, where it can be seen the geometrical shape to understand the steps that below are illustrated.

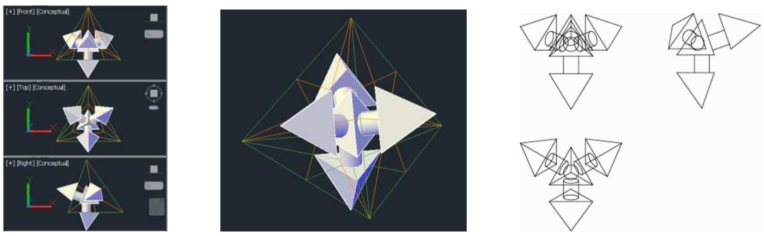


Fig. 6 Structural node: (a) Render, (b) Orthogonal projection.

As we can observe in Figures 7 and 8, the structural node was built in four steps, in the first three steps were traced in the center of each of the faces of the tetrahedron two, the tetrahedrons four, five, six and seven of the same magnitude of the tetrahedron three.

In the fourth step were traced, the cylinders of 10 mm. in diameter, taking into account the center of the faces that are parallel to the faces of the tetrahedron two to form a single shape.

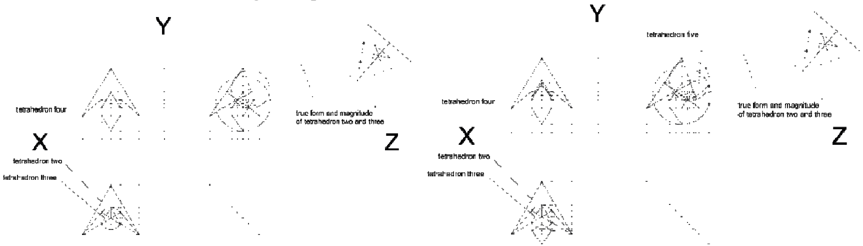


Fig. 7 Orthogonal projection: (a) Tetrahedron four, (b) Tetrahedron five.

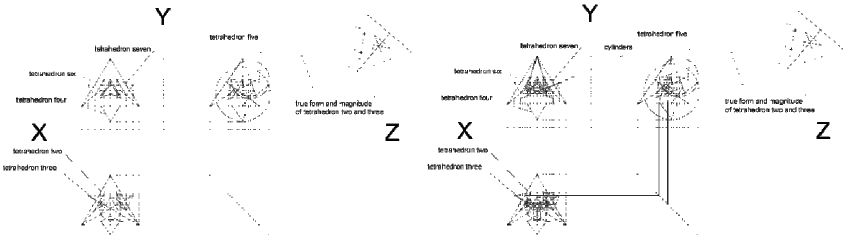


Fig. 8 Orthogonal projection: (a) Tetrahedrons six and seven, (b) Cylinders.

Before proceeding with the construction of the three-dimensional structure that is presented in the following section, it is very important that we analyze the location and distance of the axes linking the cylinders with the tetrahedrons and the resulting angles of each of the intersections. As shown in Figure 9, the resulting angles are 19 °, 71 ° and 90 ° respectively.

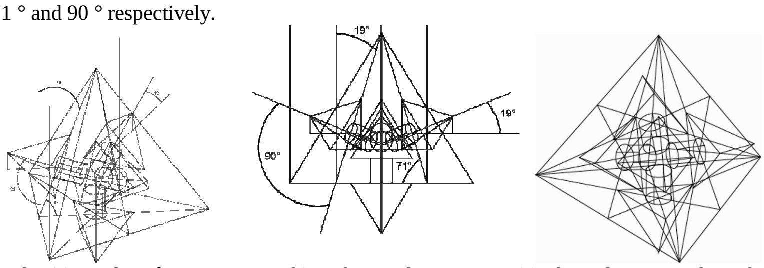


Fig. 9 Structural node: (a) Angles of intersections, (b) Orthogonal projection, (c) Three-dimensional graphical representation

Taking into account the center of the tetrahedron three whose origin is (0,0,0,) the location of the axes linking the cylinders with the tetrahedrons are:

	Axis one			Axis two			Axis three			Axis four		
	x	y	z	x	y	z	x	y	z	x	y	z
start	0	0	0	0	0	0	0	0	0	0	0	0
end	0	0	-41.24	33.67	19.67	13.74	-33.67	19.44	13.74	0	-38.88	13.74
delta	0	0	-41.24	33.67	19.67	13.74	-33.67	19.44	13.74	0	-33.88	13.74
length	41.24	41.24	41.24	41.24	41.24	41.24	41.24	41.24	41.24	41.24	41.24	41.24

5. Three-dimensional structure

The three dimensional structure [3] is composed of five nodes, four in each of the vertices and the fifth in the center of gravity of the tetrahedron one, thus achieving the balance of the whole structure. To join each of the nodes of the vertices are used cylindrical shapes, and to link the nodes of the vertices with the center node, are used prisms of three sides.

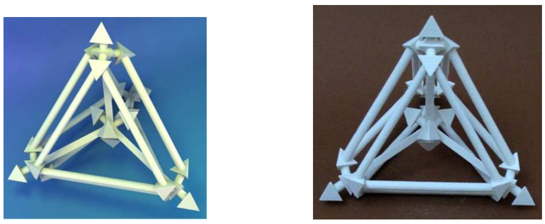


Fig. 10 Three-dimensional structure (a) Render, (b) Prototype.

As shown in Figure 11, to construct the three dimensional structure, first are rotated the nodes located at the vertices so that three of its faces are parallel to the faces of the tetrahedron one, so that they can interconnect with the bars of triangular shape with a length of 98.2093 mm. The base of the rods is triangular because, the central tetrahedron of each of the nodes is truncated in the third part of the edge of the tetrahedron, i.e. to 9.6225 mm.

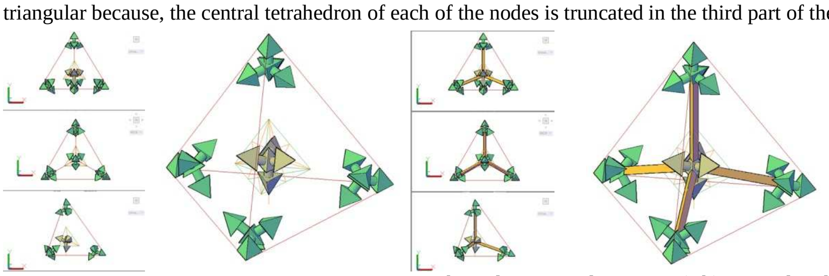


Fig. 11 Three-dimensional structure (a,b) Rotated nodes, (c,d) Triangular bars.

To join the nodes of the vertices, are used cylindrical bars of 10 mm. in diameter, which are located in the center of the faces of the tetrahedrons.

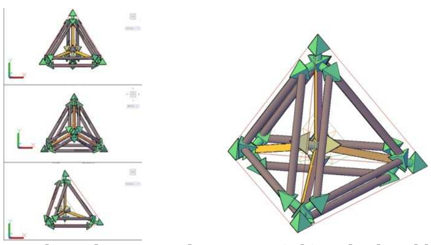


Fig. 12 Three-dimensional structure (a,b) Cylindrical bars.

If we take into account the principle that the triangle is the single polygon which does not deform when a force is applied, we may design, as shown in the Figure 12, a three-dimensional structure based on the tetrahedron where we may be able to find several triangular shapes, as shown in figure 13.

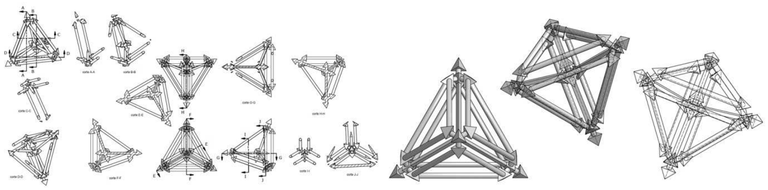
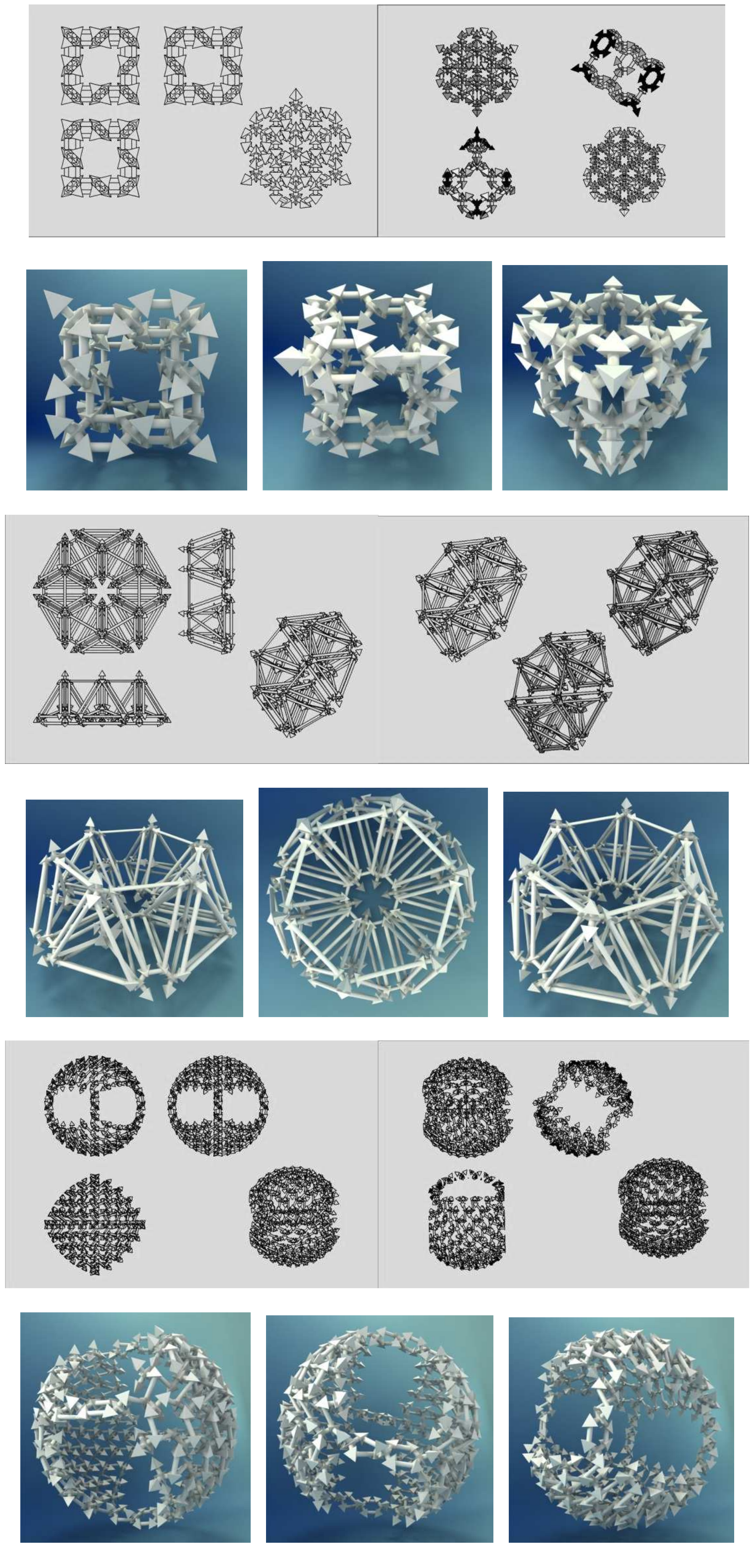


Fig. 13 Triangular shapes: (a) Orthogonal projection, (b) Three-dimensional graphic representation.

6. Results

The results obtained in the design and modeling of the “Structural node” based on the tetrahedron, led us to create a three-dimensional structure dimensionally stable to support its own weight, was obtained the transmission of a visual language through modeling structurally balanced, and was established a conceptual language to generate spatial structures as cube, a hexagonal shape and a sphere.



7. Conclusions

The design of the structural node based on the geometry of the tetrahedron, can be used to create new geometric structures that can be considered as large-scale architectural spaces without the need to place other structural elements such as columns, which divide our space.

8. References

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