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# MÖBIUS PAIRS

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The Möbius configuration is a certain configuration in Euclidean 3-dimensional space consisting of two mutually inscribed and circumscribed tetrahedra. Each vertex of one tetrahedron lies on a face plane of the other tetrahedron and vice versa. The configuration consists of 8 points (vertices) and 8 blocks (planes), such that each block consists of 4 points and each point is contained in 4 blocks.

It is known from [2], that there are five $8_4$ -configurations with the property that at most two planes share two points, and dually: at most two points are common to two planes. The configurations of interest are precisely those with two tetrahedra, each inscribing and circumscribing the other.

One may consider a configuration of this type in $n$ -dimensional space: we take two $n$ -simplices instead of tetrahedras. The obtained structure will be reffered to as a *Möbius pair* (cf. [1]) of $n$ -simplices, or shortly *Möbius $n$ -pair*. The manner of incsribing (and all construction) may be well expressed in a combinatorial approach. Let us consider two disjoint $n$ -element sets, say A and B. We identify vertices of one simplex with elements of A, and vertices of the second simplex with elements of B. Then faces of simplices correspond to the $(n-1)$-element subsets of A and B, respectively. Finally, we set some permutation $\varphi$ of $n$ -element set, that assigns vertices of one simplex to the faces of the another one. As an effect we get combinatorial configuration, denoted by $M_{(n,\varphi)}$ .

It was proved in [4], that the classical Möbius configuration ( $M_{(4,id)}$ ) may be realized in a 3-dimensional projective space if and only if the basic field is commutative. In [1], the existence of non-degenerate Möbius pairs is established for projective spaces of all odd dimensions.

The number of pairwise nonisomorphic configurations $M_{(n,\varphi)}$ coincides to the number of conjugacy classes of the group $S_n$ . We set conditions, under which $M_{(n,\varphi)}$ contains Möbius $n$ -pairs distinct to the original Möbius $n$ -pair, that in fact defines $M_{(n,\varphi)}$ . Moreover, it turns out, that for $k < n$ Möbius $k$ -pairs may be nested in $M_{(n,\varphi)}$ .

From the original paper of Möbius [3], the automorphisms group of $M_{(4,id)}$ has order 192. We characterize automorhisms of the remaining structures $M_{(n,\varphi)}$ and establish corresponding groups.

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