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## THE SLICE-BASED TRANSFORMATIONS IN AUTOCAD

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In classical Descriptive Geometry we can find constructions a such of geometric surfaces as: twisted column, the Saint-Gilles surface and canal surface with relation to a helix as a path [1] (Fig. 1). This leads to the concept of the so-called slice-based transformation [2]. Many of the simple and frequently used nonlinear transformations operate as follows. One slices the object to be transformed by parallel planes. These slices are then rearranged (displaced, and perhaps also scaled) in a systematic way to a new object.

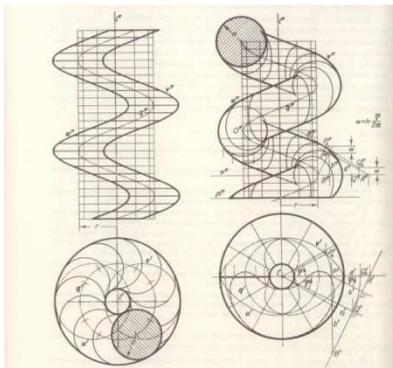


Figure 1: *Twisted column* - supporting plane of generating circle orthogonal to helix axis (left), *Saint-Gilles surface* - circle in meridian plane (right) [1]

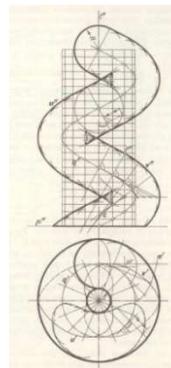


Figure 2: *Canal surface* - supporting plane of generating circle orthogonal to helix tangent [1]

The twist deformation [2] will be discussed first. To define the *twist* deformation, we select a fixed bottom plane  $\beta$  and a straight line  $a$  (called *twist axis*)

orthogonal to the plane  $\beta$  (Fig. 8b). The layers of the object in planes orthogonal to the axis (i.e., parallel to  $\beta$ ) are now rotated about  $a$  as follows. The bottom plane  $\beta$  remains fixed and the rotational angle  $\alpha_{max}$  of the top plane  $\tau$ , is prescribed. We assume that the distance between the bottom and top planes is  $h$ , the height of the object to be deformed. The rotational angle  $\alpha(z)$  of the slice at height  $z$  above  $\beta$  is selected to be

$$\alpha(z) = \frac{z}{h} \alpha_{max} \tag{1}$$

There is an obvious connection with helixes and helical motions. Any straight line parallel to  $a$  is mapped to a helix with axis  $a$ . Another interesting property of the twist operation is that it *preserves the volume*.



Figure 3: The Turning Torso (2000-2005) in Malmo by Santiago Calatrava [2,3]

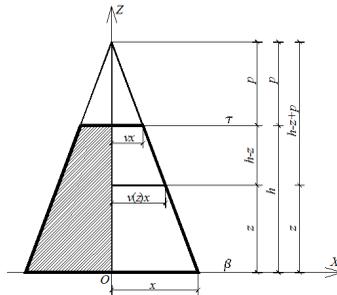


Figure 4: Assumptions for the derivation of tapering transformation formula

Tapering deformation [2]. To describe the *tapering* deformation in a precise way, we use a mathematical description and select an adapted Cartesian coordinate system in which  $a$  is the  $z$  axis and  $\beta$  is the  $Oxy$ -plane. Furthermore, we assume that the scaling directions are parallel to the  $Ox$  - and  $Oy$  - axes, respectively. Hence, in the top plane  $\tau$  (which has the equation  $z = h$  ( $h$  being the distance between  $\beta$  and  $\tau$ )) the scaling is of the form  $x' = v \cdot x$ ,  $y' = w \cdot y$ . Here, we have denoted the user-specified scaling factors in the  $x$  and  $y$  directions by  $v$  and  $w$ , respectively. Because the bottom plane remains fixed, we may say it is scaled with factor 1 in both directions. One then uses a linear variation of the scaling factors between bottom and top planes. This means that the scaling factor  $v(z)$  for the  $x$  direction in height  $z$  is equal to

$$v(z) = 1 + \frac{z}{h}(v - 1). \quad (2)$$

From the Figure 4 we can write

$$\frac{x}{h+p} = \frac{vx}{p} \rightarrow p = \frac{vh}{1-v}. \quad (3)$$

Using the denotations from the Figure 4

$$\frac{v(z)x}{p+h-z} = \frac{v(z)x}{p} \rightarrow pv(z) = v(p+h-z) \quad (4)$$

and by substituting (3) in (4) we can derive of the formula (5):

$$\frac{vh}{1-v}v(z) = v\left(\frac{vh}{1-v} + h - z\right) \rightarrow v(z) = 1 + \frac{z}{h}(v - 1). \quad (5)$$

As desired,  $v(0) = 1$  and  $v(h) = v$ . Replacing  $v$  with  $w$ , we obtain the scaling factor in the  $y$  direction. In the plane at height  $z$ , the scaling is  $x' = v(z) \cdot x$ ,  $y' = w(z) \cdot y$ . Inserting the expressions (5) for the scaling factors, we arrive at the following analytical representation of the tapering operation:

$$\begin{aligned} x' &= x + \frac{xz}{h}(v - 1) \\ y' &= y + \frac{yz}{h}(w - 1) \\ z' &= z. \end{aligned} \quad (6)$$

Because we see on the right-hand side terms  $x \cdot z$  and  $y \cdot z$ , respectively, tapering is not a linear transformation. It is a special *quadratic* transformation.

If one wants to have thickening or thinning effects without loss of smoothness, one can use a deformation technique sometimes called *bulge*. It works like tapering, but uses smooth scaling functions  $v(z)$  and  $w(z)$ .

**Shear deformations** [2]. Twisting rotates the slices. Tapering and bulging scale the slices. *Shear* deformations apply translations to the slices. Because a translation is determined if we know the image point of a single point, one prescribes the image  $a'$  of the axis  $a$ . The shear deformation is determined uniquely by the change of the axis. Shear transformations leave the shape and thus also the area of the slices unchanged. They also preserve the height  $z$  of each slice. By Cavalieri's principle, shear transformations are therefore volume preserving. Architecture that may be the result of a shear-like deformation is realized in London (City Hall by Norman Foster).

**Bending deformations** [2]. Some systems may contain the nonlinear shear deformations discussed previously as versions of *bending*. In the following we

examine bending deformations that also change the orientation of the slices. In an elementary *radial bending* operation, the axis  $a$  is mapped to a circle  $a'$  and the slices are rearranged in planes normal to the circle.

There are also modeling options that changed to  $a$  more general curve (e.g., a Bézier curve) and rearrange the slices in planes normal to  $a'$ . The arising freedom of a rotation about  $a'$  may be resolved as in our discussion of the rotation-minimizing frame.

The way of the realization the discussed above transformations is various in AutoCAD: indirect or direct depending on complexity of the transformation form:  $a \rightarrow a'$ .

The remainder of the paper presents the implementation of the previously discussed surface and the "simple transformation" in AutoCAD, version 2012.

Implementation of the following surfaces: twisted column, Saint-Gilles surface, and channel surface can be obtained in several ways. One can use the SWEEP command with the option of Alignment (Fig. 5).

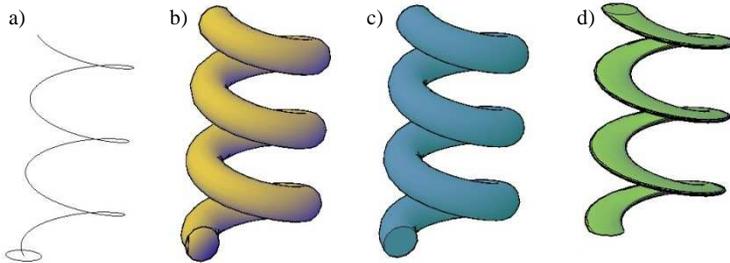
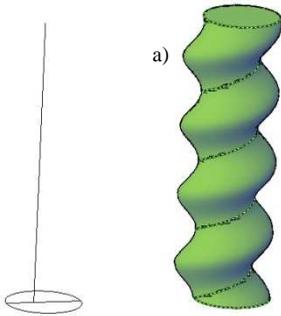


Figure 5. Implementation of the surface from classical geometry: a) path and cross-section; b) the surface of Saint-Gilles; c) channel surface; d) the surface of twisted column (SWEEP with the option of Alignment)

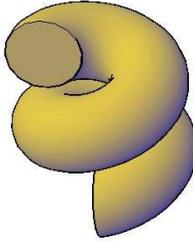
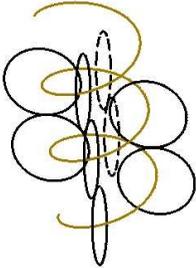


a)

b)

Unfortunately, the SWEEP function has its limitations when it comes to dragging cross sections. We are not able to model an object when the size of its cross sections of the result will interpenetration of the object. In particular, it is impossible to generate a classical architectural twisted column by dragging the cross sections along the helix.

However, the twisted column can be obtained when we drag the cross section (circle) along a straight line (vertical),

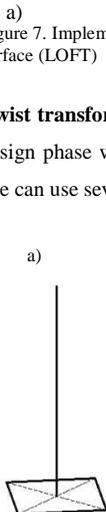


simultaneously rotating it about the point which is not the center of the circle (Fig. 6).

Figure 6. Implementation of a real architectural element - a twisted column

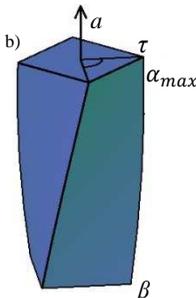
column: a) path and cross section; b) the surface of the twisted column (SWEEP with a rectilinear path and rotation about the point which is not the center of the circle)

Another way to obtain these surfaces is to use the LOFT command. This is an approximate solution, however, often fully satisfactory (Fig. 7).



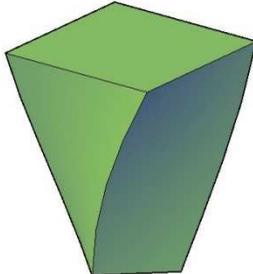
a)

Figure 7. Implementation of the Saint-Gilles surface: a) path and cross-sections; b) the Saint-Gilles surface (LOFT)



b)

c)



**Twist transformations.** We cannot turn, bend, etc. the created object. Already in the design phase we need to know the parameters that will allow to get the final result. We can use several functions. These are the commands: SWEEP, LOFT (Fig. 8, 9).

Figure 8. Illustration of the twist transformation: a) path and cross-section; b) the obtained solid; c) the obtained solid with scaled top base (scale 2:1) (SWEEP)

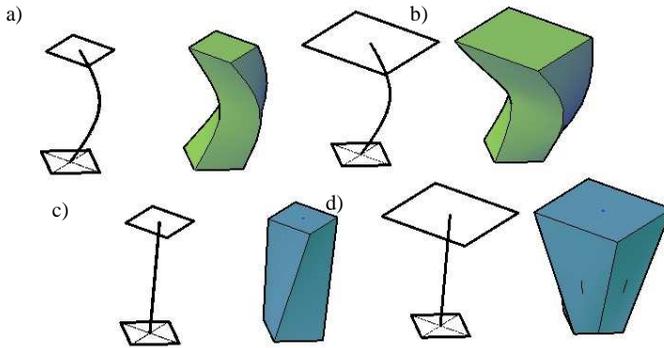


Figure 9. Implementation of the twist and tapering transformations: a) the curvilinear path, the two cross sections and the obtained solid; b) a curvilinear path, the two sections with scaling, and the obtained solid; c) vertical path, the two cross sections and the obtained solid; d) vertical path, the two sections with scaling, and the obtained solid (LOFT)

**Bending transformations.** Bending transformations are the simplest to obtain, we create them by using the REVOLVE function. The required solid is obtained by specifying the cross section and the axis or path about which we revolve our cross section (Fig. 10).

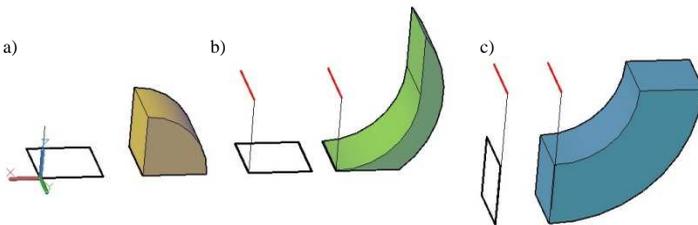


Figure 10. Implementation of bending transformations: a) rotation of the cross section about the axis of  $Oy$ ; b) rotation about the path, cross section parallel to the  $Oxy$  plane; c) rotation about the same path as in the case of b, but the cross section is parallel to the  $Ozy$  plane (REVOLVE)

**Tapering transformations.** We can obtain tapering transformations in many ways using the following commands: EXTRUDE, SWEEP, LOFT (Fig. 11, 12).

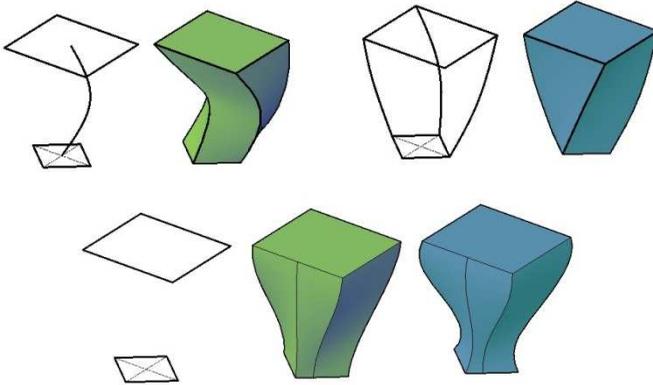


Figure 11. Implementation of the tapering transformations (LOFT)

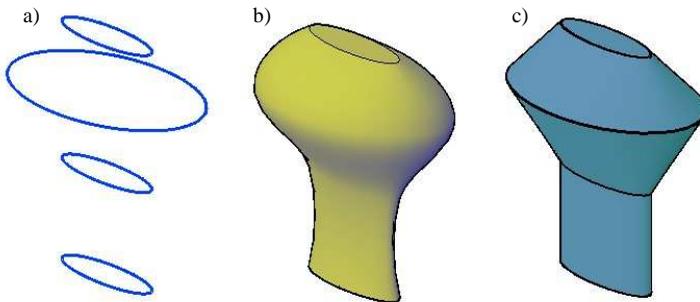


Figure12. Implementation of tapering transformations: a) cross sections; b)connection by means of smooth surface; c) connection by means of ruled surface (LOFT)

**References:**

- [1] Otto F., Otto E.: *Podręcznik geometrii wykreślnej*. Wydawnictwo Naukowe PWN, Warszawa, 1994.
- [2] Pottmann H., Asperl A., Hofer M. and Kilian A. (2007). *Architectural Geometry*. Bentley Institute Press. ISBN 978-1-934493-04-5.
- [3] [http://www.ask.com/pictures?qsrc=167&o=101699&l=dis&q=turning%20torso&locale=en\\_US](http://www.ask.com/pictures?qsrc=167&o=101699&l=dis&q=turning%20torso&locale=en_US)