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GEOMETRIC COMPACTNESS INDICATORS OF A BUILDING

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In the paper [3], analyzing the geometry of solid of the building introduced the concept of geometric compactness as "quotient of external walls area and volume of the building". This indicator shows how much building material should be used to obtain a specific cubature of the building. But it is also appointed value and depends on the adopted unit of measurement.

In this paper will be proposed various ways of determining the *indicator of relative geometric compactness of the solid*.

The relative indicator of the geometric compactness of solid with relation to cube. The first indicator will be presented the *relative geometric compactness of the solid relative to cube*. It is defined in the following way. By implication we consider account the building (i.e. angular solid) of the rectangular polygon and measure its base area *A* (projection, building area), perimeter *p* and volume (cubature) , wall surface area (external) and the total surface area $CA =$. Ratio GC (Geometric Compactness of Solid), is commonly compactness, but expressed in units [1/m].

Consider now a cube with edge length equal *a*, geometric compactness of cube expressed as the quotient of

$$GCC = \frac{6a^2}{a^3} = \frac{6}{a} \qquad (1)$$

Now assume that shape of the building with a cubature is cubic and we determine the cube edge length. We have and thus $\sqrt[3]{A \cdot h}$. Then the surface area of a cube is expressed in the form of CA^*. So, the geometric compactness of such a cube is equal to

$$\frac{CA^*}{V^*} = \frac{6}{(A \cdot h)^{\frac{1}{3}}} \qquad (2)$$

Hence, for , we can determine the quotient of the areas (total) of the cube to the solid of the considered building. Then the resulting quotient of

$$RC = GCC : GCS = \frac{6(A \cdot h)^{\frac{2}{3}}}{V^*} : \frac{p \cdot h + 2A}{V} = \frac{6(A \cdot h)^{\frac{2}{3}}}{p \cdot h + 2A} \qquad (3)$$

is relative compactness of solid relative to the cube. The quotient (3) can be regarded as compactness of a prism on the area *A*, perimeter *p* and height *h* relative to the cube. It is the relative geometric compactness *RC* of building, which is said in the literature [2].

The compactness of the rectangular solid with base of a square and height equal height of the solid. Consider now a cuboid with a square base with edge lengths equal *a* and height equal *h*, geometric compactness of cuboid expressed as the quotient of

$$GCCd = \frac{2a^2 + 4a \cdot h}{a^2 h} = \frac{2a + 4h}{ah} \qquad (4)$$

Suppose the solid of the building with a cubature is a prism of any basis and we determine the length of the edge of the square of the cuboid base.

We have a and thus a. Then the cuboid area $CAC^* = 4$. Then

$$\frac{CAC^*}{V^*} = \frac{4A^{\frac{1}{2}}h + 2A}{A \cdot h} = \frac{4h + 2A^{\frac{1}{2}}}{A^{\frac{1}{2}} \cdot h} \qquad (5)$$

Hence, for , we can determine the quotient of the areas (total) of the solid cuboid to the prism of any base (considered building). Then we obtain the quotient (Relative Compactness of solid with relation to Cuboid)

$$RCC = GCCd:GCS = \frac{4A^{\frac{1}{2}}h + 2A}{V^{*}} : \frac{p \cdot h + 2A}{V} = \frac{4A^{\frac{1}{2}}h + 2A}{p \cdot h + 2A} \tag{6}$$

For a cuboid with dimensions *a, b, h* we obtain

$$RCC = \frac{4(a \cdot b)^{\frac{1}{2}}h + 2a \cdot b}{2(a+b) \cdot h + 2a \cdot b} \tag{7}$$

The quotient (7) can be regarded as compactness of cuboid with any base of area *A*, perimeter *p* and height *relative to the cuboid with a square base and height h*

Geometric compactness measured by compactness of base of solid. We can finally resign from a height of a building and base area of solid: top and bottom. Hence the optimization will only include area of shape of the base (plan of the building). Then geometric compactness of base (referenced square) expressed as the quotient of

$$GCS = \frac{4a}{a^2} = \frac{4}{a} \tag{8}$$

While the compactness of the figure base on area *A* and perimeter *p* expressed as a quotient . Since , we have a :. Than (8) takes the form

$$GCS = \frac{4}{\sqrt{A}} \tag{9}$$

Hence, for , we can determine the quotient of the areas (base) square to the base (considered building). Then we obtain the ratio (Relative Compactness of solid with relation to Base)

$$RCB = GCS:GCR = \frac{4}{\sqrt{A^{*}}} : \frac{p}{A} = \frac{4A^{\frac{1}{2}}}{p} \tag{10}$$

for a rectangle of *a, b* we obtain

$$RCB = \frac{2(a \cdot b)^{\frac{1}{2}}}{a+b} \tag{11}$$

The relative indicator of the geometric compactness of solid relative of the sphere. Consider now a sphere of radius length *r*, geometric compactness of sphere expressed as the quotient of

$$GCSph = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r} \tag{12}$$

Now assume that shape of the building with a cubature is sphere and we determine the length of sphere radius. We have $\frac{4}{3}\pi$ and thus

$r =$. Then geometric compactness of sphere expressed as the ratio of

$$GCSph = \frac{3}{\left(\frac{3}{4\pi}A \cdot h\right)^{\frac{1}{3}}} \tag{13}$$

Hence, for , we can determine the quotient of the areas (base) of sphere to considered solid of the building. Then we obtain the quotient of

$$RCS = GCC:GCS = \frac{4\pi\left(\frac{3}{4\pi}A \cdot h\right)^{\frac{2}{3}}}{V^{*}} : \frac{p \cdot h + 2A}{V} = \frac{(36\pi)^{\frac{1}{3}}(A \cdot h)^{\frac{2}{3}}}{p \cdot h + 2A} \tag{14}$$

which is the relative compactness of solid relative to the sphere. The quotient (14) can be regarded as *compactness of solid* of prism with base of area *A*, perimeter *p* and height *h* calculated *relative to the sphere*. It is the *relative geometric compactness RC* solid (building), which is said in the literature [2]. Indicator in formula (14), approximately equal to 4,84, it expresses the ratio of the area surface of the sphere to its volume, with the assumption, that volume is equal 1. Similarly, the number 6 in formula (3) expressed ratio of area of cube to its volume, with the assumption, that volume of cube is equal 1. The compactness of the sphere is therefore larger (ratio equal 4,84 - smaller) than cube (ratio equal 6 - larger). For purposes of illustration these values for the various Platonic solids are equal: for tetrahedron – 7,21; cube – 6; octahedron – 5,72; dodecahedron – 5,31; icosahedron – 5,148; sphere – 4,836. Sphere is therefore the most compact („economical in building material") solid.

To assess the compactness of buildings from among the Platonic solids we choose a cube, because of his right angles walls. Indicators of geometric compactness of solidswith relation to the cube have a universal and unequivocal character. But do not give an objective assessment of ratio of the compactness of the solid (the building). The proportion of compactness show only indicators (*RCC*) relative compactness of a rectangular building with relation to a square base.

Indicators of compactness of solids whose bases are rectangular polygons. Next we take care rectangular polygons [1] inscribed in a rectangle (Fig. 1). We will say, that the rectangular polygon W_P (with sides $x_1^b,\ldots,x_q^b,x_1,\ldots x_r, x_1^t,\ldots,x_s^t, y_1^l,\ldots,y_l^t,y_1,\ldots,y_u, y_1^r,\ldots,y_w^r$) is *inscribed in a rectangle*

P (with sides x^b, y^r, x^t, y^l) if and only if and each side of the rectangle P contains at least one side of the polygon W_P. By not considering degenerate polygons, and have quite broad class of figures, we assume that the reported polygons are simply connected, and generally multi-connected (can have holes) (Fig. 1c). Connectedness of the polygon (generally determined for a flat collection) means that the any two points of the polygon can be connected polyline (polygonal line [4]) lies inside the polygon. Connectedness of polygon W_P inscribed in a rectangle P means, that every line parallel to the sides of the rectangle P, containing the points inside the rectangle contains the interior points and boundary points of the polygon W_P (Fig. 1). Then every point of the edge of the rectangle P is the projection of at least two points of the boundary polygon W_P. So we can use the sides of the polygon W_P (after parallel moving respectively to the axis *OX*, *OY* at the edge of the rectangle P) „to wallpaper" edge of the rectangle P (Fig. 1a). Some parts of the „wallpaper" will impose on itself several times (Fig.1b,c). Thus it easily follows that the length of the perimeter of the polygon W_P is larger than or equal to the length of the perimeter of the rectangle P. In some cases it may be higher, and then to analysis of shapes will be useful to know the difference in length of perimeter of these figures.

The relative defect of perimeter in the rectangular polygon. If we denote by perimeter of region F with relation to the polygon W_P inscribed in a rectangle P, we can write

$$p(W_P) = p(P) + \Delta p(W_P) \tag{15}$$

where

$$\Delta p(W_P) = \sum_{i=0}^{n-1} 2i \cdot x_{2i+2} + \sum_{j=0}^{m-1} 2j \cdot y_{2j+2} \tag{16}$$

but means the sum of measures orthogonal projections to the axis *OX* of all parts of the sides $x_l^b(x_k^t)$ polygon W_P parallel to the axis *OX*, that straight line parallel to the axis *OY* intersect in total 2*i* points, with *i*=0,1,…,*n*-1 (Fig. 1a), means the sum of measures orthogonal projections to the axis *OY* of all parts of the sides y_l^l (y_k^r) polygon W_P parallel to the axis *OY*, that straight line parallel to the axis *OX* intersect in total 2*j* points, with *j*=0,1,…,*m*-1 (Fig. 1b,c). Numbers 2*n* (2*m*) indicate the maximum number of sides of polygon W_P parallel to axis *OX* (*OY*), that crosses the line parallel to the axis *OY* (*OX*). Quantity , defined by the formula (15), will be called *perimeter defect* of a polygon W_P. Determined by formulas (15), (16) perimeter defect polygon W_P depends on a unit of measurement of length and, thus does not give an unambiguous measure of the difference, between the perimeter of the rectangular polygon and rectangle. Hence the rational is to determine the *relative perimeter defect* of the polygon W_P as the quotient of

$$\delta_p(W_P) = \frac{\Delta p(W_P)}{p(P)} \tag{17}$$

and therefore not appointed a unit of measurement deviation in length of the rectangular perimeter of the polygon described on the rectangle. From an economic and ecological (energy saving) point of view, we have when the relative perimeter defect of building design solution takes values close to zero. But it remains an open volume of "loss" caused by a possible reduction in the surface area of a rectangular polygon, so the building cubature. This volume "loss" reflects another indicator related to the area of the polygon.

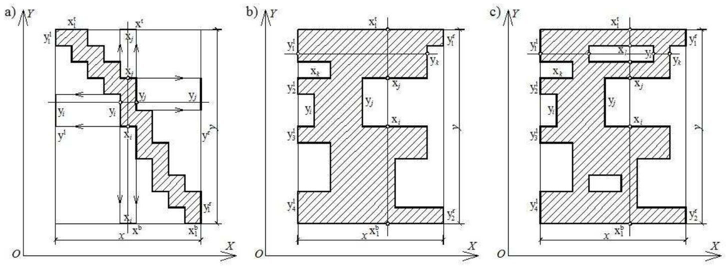

Fig. 1: Rectangular polygons inscribed in a rectangle P (with sides x^b, y^r, x^t, y^l) with dimensions $x{\times}y$

The relative area defect of the rectangular polygon. The second parameter characterizing the geometry of a rectangular polygon W_P is his area . Area of rectangular polygon can be expressed as

$$\delta a(W_P) = a(P) - \Delta a(W_P) \tag{18}$$

where can be called *area defect* of a rectangular polygon. Then the quotient

$$\delta a(W_P) = \frac{\Delta a(W_P)}{a(P)} \tag{19}$$

called a *relative area defect* of rectangular polygon W_P in relation to that described in the rectangle P. This indicator expresses the percentage of "loss" of the surface area (the building cubature) in relation to the area of rectangle, and while keeping the size of perimeter of the rectangular polygon (such as in a normal rectangular polygon [4]) is part of "loss" of the construction on the given resources consumption of building materials in the construction of the object and the demand for energy during using the building.

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