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USING A CAS TO VISUALIZE SOME IMAGES OF LINES MAPPED VIA THE HARMONIC CROSS-RATIO

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Among transformations of the three-dimensional projective space P^3 onto itself there is the mapping constructed with the use of the basic curve C^3 and the bisecants forming the line congruence of the first order and third class. A particular case of this transformation takes place when an object under transformation is a line crossing the basic curve C^3 at exactly one point. This paper concerns a computer visualization of this particular situation. Such a visualization can be done within computer algebra systems because they provide exact computation. Thanks to this feature one can show, in the graphical form, several interesting properties of the map at hand. In the paper we produce, using the computer algebra system Derive 5 from Texas Instruments, Inc., images of lines; these images are conics. An example of the visualization we talk about is seen in Fig.1 and Fig.2. Both figures are produced when the line under the (1, 3)-line congruence passes through points (1 : 1 : 1 : 1) and (3 : 2 : 2 : 3). In Fig.1 below there is traced, in the cube $\langle -0.5, 0.5 \rangle^2$, the image of this line, this is the hyperbola covered by the equation

$$x = -q'(3u + 7), y = -q'(2 + u), z = q'(u + 2),$$

where $q := 3 \cdot (6u^2 + 11u + 10)$,

$$u \text{ runs the intervals } (-\infty, z_1) \cup (z_1, z_2) \cup (z_2, +\infty),$$

$$z_1 := (-11 - \sqrt{41})/4, \quad z_2 := (-11 + \sqrt{41})/4.$$

In Fig.2. there are traced the projections of this hyperbola onto the principal planes of the Cartesian space $Oxyz$, they are shown in the Cartesian system Ohv , and we have the line marked as yz (the projection onto the plane $x = 0$; its equation is $v = -h$) and the hyperbolas marked as xy and xz (the projections onto the plane $y = 0$ and $z = 0$, resp.). On the first of these hyperbola there are put some points, the accompanying number is the value of u producing the point. When u tends to $-\infty$, as well as when it approaches $+\infty$, the points (x, y, z) of the image runs to $(0, 0, 0)$ and the projection of this point is also marked on Fig.2.

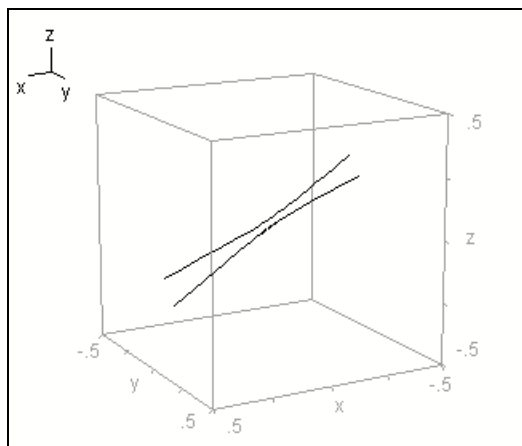


Fig.1. Arcs of the hyperbola laying in the cube $\langle -0.5, 0.5 \rangle^2$

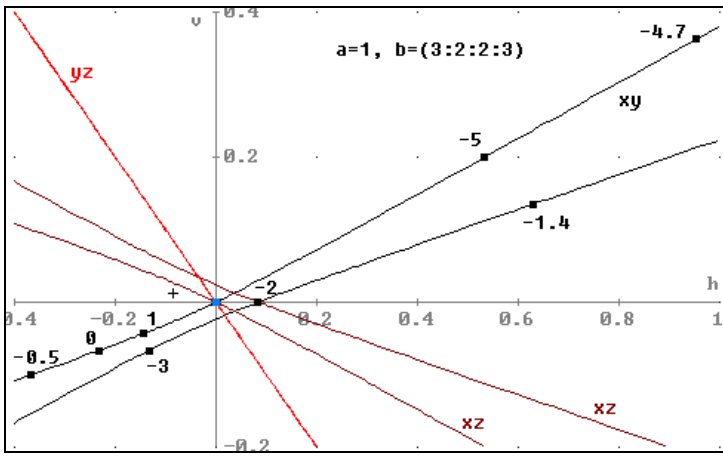


Fig.2. Arcs of curves $xy: h = -q \cdot (3u + 7), v = -q \cdot (2 + u)$; $yz: h = -q \cdot (2 + u), v = q \cdot (u + 2)$; $xz: x = -q \cdot (3u + 7), z = q \cdot (u + 2)$;
 moreover, eight points on the curve xy and the image of the infinity
 (this image coincides with the origin O of the Cartesian system Ohv)