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GEOMETRIC MODEL OF A SPECIALLY CREATED EMBANKMENT

Key words: *skeleton of roof, straight skeleton, parabola, Voronoi diagram for polygon, internal angle of friction, embankment*

The starting point for consideration is, in a sense, a simple experiment. In the laboratory, we cut a cardboard polygon, which can be considered as a base model of the roof (of a special polyhedral surface) [2]. We then place a cardboard at a certain height over a flat surface polygon. Next, flour or any other granular material is poured over the cardboard polygon (Fig. 1). The mechanics of granular materials indicates a material sprinkled in one spot form a heap is placed in the heap, which takes the shape of a cone. The angle at the base of the cone is called the *angle of internal friction*. This angle depends on the type of material and is an important parameter characterizing the material mechanic property. In this case the material is placed in a pile, which is similar to the roof with the constant slope [2]. But there is a difference, because in the neighborhood of the vertex concave angle of the base a curved surface is created and the ridge line of the roof is a curved line.

Figure 1: A pile of granular flour (the polygonal cardboard base with two concave angles)

By preserving the angle of internal friction, the ground under the force of gravity slides in perpendicular direction to the edge of the base. Let's create a geometric model of the pile.

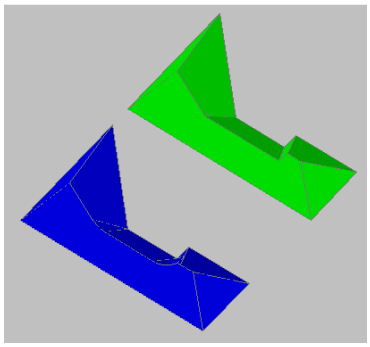

Figure 2: The virtual model of granular material pile (at the bottom), at the top - the model of a roof with the same base (made in AutoCAD program)

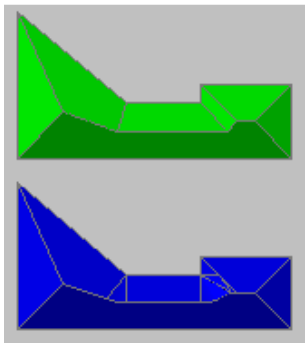

Figure 3: The horizontal projection of a virtual model of a pile of granular material (at the bottom), at the top - the model of a roof with the same base (made in AutoCAD program)

Straight lines converging at the vertices of concave angles, form a right circular cone. Simple reasoning shows that some parts of the ridge-line are parabolas. Indeed, the generatrices of a cone with the apex angle of 90 - φ at the vertex of a connected concave base form an angle φ with the plane of the base. Similarly, all plane faces of the model form an angle φ prism with a plane base. Therefore, each face is parallel to one (and only one) *generatrix* of a cone and according to the Dandelin's theorem it cuts the cone surface in the form of a parabola. Figures 2 and 3 show geometrical models of piles of granular material obtained in the experiment (made in the CAD environment).

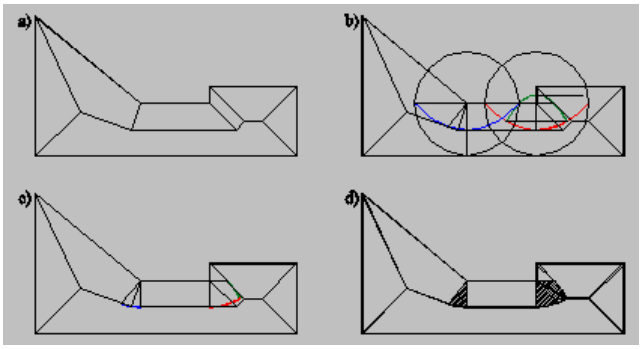

Figure 4: Roof skeleton and Voronoi diagram: a) horizontal projection of a roof; b) transformation of a roof to Voronoi diagram for a polygon; c) selected sections of the parabolas; d) Voronoi diagram for a polygonal base with the hatched parabolic areas

The horizontal projection of a roof model and a model of granular material pile are well-known geometric objects: *straight skeleton* (*roof skeleton* or briefly a *roof*) and *Voronoi diagram for a polygon* [1, 2].

References:

[1] Aichholzer O. i in.: *A Novel Type of Skeleton for Polygons*. Journal of Universal Computer Science, Vol. 1, No. 12, 1995, 752-761.

[2] Koźniewski E.: *Geometria dachów. Teoria i zastosowanie*. Wydawnictwo Politechniki Białostockiej, Białystok 2007.