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MULTIPLE REFLECTIONS/INTER-REFLECTIONS IN RELATION TO THE SPHERE

Multiple reflections/Inter-reflections in relation to the sphere constitute one the cases of *Alhazen's Problem, Alhazen's Optical Problem or Billiard Problem*.

The author of this paper defined and solved this problem using the methods of descriptive geometry and in this particular scope one of the cases of Apollonian curves of the third degree with the property defined as isoptic – *Apollonian isoptic cubic curve* for which the author provided a new method of generating it.

Thanks to the properties of this curve is is possible to determine the points of reflection in relation to the sphere in the central-reflective projection for any arrangement of points in space sphere and the centre of projections. One of the possible arrangements consists in an arrangement for which a point in space *A* and the projection centre *S* are situated in the inner space of the sphere (or a circle). It is a case in which the reflection occurs only in relation to the inner side of the sphere (or a circle). In physical terms it corresponds to the reflection in the spherical concave mirror. The course of the Apollonian curve indicates points Af on the sphere for which the reflection law is met. However, it is easy to demonstrate that there are areas of intraspherical space (inside a circle), which are restricted with a pencil of spheres with the shared centre *O* with the reflectional sphere on which it is possible to circumscribe regular n-angles (which contain on their sides a pair of points *A* and *S*) in the plane determined by three points *A, O, S* for which n assumes any value from the set (3…,∞). The vertexes of these n-angles belong to the reflection sphere (reflection circle) and they are points of multiple reflections / inter- reflections.

It should be remarked that when n = ∞ the regular polygon joins together with the circle circumscribed on it because the circle is a reflection circle and hence on the n vertexes of this polygon there is an infinite number of reflection points A_f.

When n = 3, the regular polygon is an equilateral triangle. The difference between the radii of a circle of the reflection sphere and a circle inscribed within a triangle is the biggest and comes to 2/3 of the radius of the sphere. In the case of the n-angle when n = ∞ the difference of radii is 0. It should be stated that in the case of an infinite number multiple reflection/inter-reflection the pair of points *A* and

S belongs to the reflection sphere (reflection circle) and the image of any given point A is identical with the image of the whole reflection circle.

This work is a contribution to a reply to questions concerning multiple reflections/ inter-reflections in the scope of a general problem of spherical reflections – *Alhazen's Problem, Alhazen's Optical Problem* and *Billiard Problem*. It determines the scope of discussions about this issue on the grounds of descriptive geometry and provides answers to questions resulting from defined by the author problem of multiple reflections/inter-reflections and possible deliberations resuting from them.

On the gounds of descriptive geometry, except for the exception known to the author, this problem has not been taken up yet.