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HERMITE CURVES WITH GIVEN CURVATURE AND GENERALIZATION FOR SURFACES

It is a widely-known and simple method to extend the basic idea of Hermite interpolation to polynomials of higher degree. The starting point of this paper is the degree-5 Hermite Interpolation.

The normal way of defining the curve is giving the endpoints, the two extreme tangent vectors, and the two extreme second derivates. If we note these geometrical dates by row vector $\mathbf{G} = [\mathbf{G}_1 \quad \mathbf{G}_2 \quad \mathbf{G}_3 \quad \mathbf{G}_4 \quad \mathbf{G}_5 \quad \mathbf{G}_6]$, then the curves' expression is the following: $\mathbf{Q}(t) = \mathbf{GMT}$, where $\mathbf{T} = [t^5 \quad t^4 \quad t^3 \quad t^2 \quad t \quad 1]^T, t \in [0,1]$, and the **M** is 6x6 type real matrix, which is easy to calculate.

In this paper we will show a simple calculation to determine the two extreme second derivates, if the Osculating circles are given in the points. By this method it is possible to construct an interpolate curve for given points, given tangent vectors and curvatures (Osculating circles) in each point. Obviously in the connection points the segments will have second order continuity and in practice this method can be more practicable for designers than the original one.

In the second part I will show how it is possible to apply the results for surfaces. The general form of such type of tensor-multiply surfaces is the following:

$$\mathbf{r}(u,v) = \mathbf{U}^T \cdot \mathbf{M}^T \cdot \mathbf{G} \cdot \mathbf{M} \cdot \mathbf{V},$$

where $\mathbf{G}$ is a 6x6 matrix which stores the geometrical data of the surface.

$$\mathbf{G} = \begin{bmatrix} \mathbf{r}(0,0) & \mathbf{r}(0,1) & \dot{\mathbf{r}}_v(0,0) & \dot{\mathbf{r}}_v(0,1) & \ddot{\mathbf{r}}_{v^2}(0,0) & \ddot{\mathbf{r}}_{v^2}(0,1) \\ \mathbf{r}(1,0) & \mathbf{r}(1,1) & \dot{\mathbf{r}}_v(1,0) & \dot{\mathbf{r}}_v(1,1) & \ddot{\mathbf{r}}_{v^2}(1,0) & \ddot{\mathbf{r}}_{v^2}(1,1) \\ \dot{\mathbf{r}}_u(0,0) & \dot{\mathbf{r}}_u(0,1) & \ddot{\mathbf{r}}_{u,v}(0,0) & \ddot{\mathbf{r}}_{u,v}(0,1) & \dddot{\mathbf{r}}_{u,v^2}(0,0) & \dddot{\mathbf{r}}_{u,v^2}(0,1) \\ \dot{\mathbf{r}}_u(1,0) & \dot{\mathbf{r}}_u(1,1) & \ddot{\mathbf{r}}_{u,v}(1,0) & \ddot{\mathbf{r}}_{u,v}(1,1) & \dddot{\mathbf{r}}_{u,v^2}(1,0) & \dddot{\mathbf{r}}_{u,v^2}(1,1) \\ \ddot{\mathbf{r}}_{u^2}(0,0) & \ddot{\mathbf{r}}_{u^2}(0,1) & \dddot{\mathbf{r}}_{u^2,v}(0,0) & \dddot{\mathbf{r}}_{u^2,v}(0,1) & \ddddot{\mathbf{r}}_{u^2,v^2}(0,0) & \ddddot{\mathbf{r}}_{u^2,v^2}(0,1) \\ \ddot{\mathbf{r}}_{u^2}(1,0) & \ddot{\mathbf{r}}_{u^2}(1,1) & \dddot{\mathbf{r}}_{u^2,v}(1,0) & \dddot{\mathbf{r}}_{u^2,v}(1,1) & \ddddot{\mathbf{r}}_{u^2,v^2}(1,0) & \ddddot{\mathbf{r}}_{u^2,v^2}(1,1) \end{bmatrix}$$

I will show some tools to modify the shape of the Hermite tensor-surface. The use of this method makes it possible to connect patches in second order continuity.

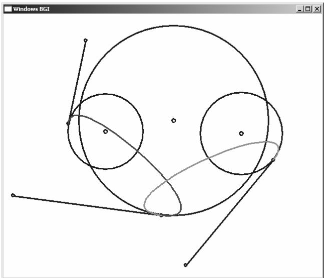

Figure 1

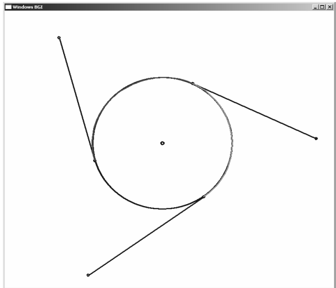

Figure 2

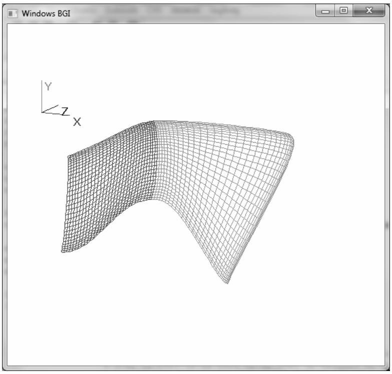

Figure 3

These figures above show some results. In Figure 1 two connecting segments are drawn. In Figure 2 three connecting segments are drawn with identical Osculating circles. In Figure 3 two connecting patches can be seen.