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SUM OF FINITE PROJECTIVE PLANES

The technique involving difference sets is one of the standard ones used to construct block designs of various types. In particular, every finite Desarguesian projective plane can be defined with the help of this method. The idea is simple – blocks (lines) are the images of some fixed subset D of a group G under left translations of this group. Some conditions may be imposed on D which assure that the resulting incidence structure is a λ-design, and such a set D is called a difference set in G. Specifically, for $\lambda = 1$ we obtain a linear space. Some weaker conditions are imposed on D, which yield a regular partial difference space – such a set D we call partial difference set.

Construction of (some kind of) sum of block designs, which are defined applying difference sets, is introduced. Roughly speaking, this construction corresponds to the operation of the direct sum of groups. The sum of designs such that every of them is given by a partial difference set D_i in a group G_i, is a block design given by some partial difference set in a direct sum of underlying groups G_i . Finite projective spaces are important class of, considered by the meaning of difference sets, configurations therefore, special attention to the sum of them is paid.

The neighbourhood of a point of the product of projective planes is analyzed. Examples of product of planes PG(2,2), and PG(2,3) given by equal, or distinct difference sets are considered, and corresponding automorphisms groups are determined. A structural characterization of the resulting configuration is made. It is proved, that every maximal projective plane, embedded in such sum, is isomorphic to the undelying projective plane. The family of maximal subplanes is a covering of the sum of finite projective planes. This covering is definable in the language of geometry of the obtained design.