Edwin KOŹNIEWSKI, Marcin ORŁOWSKI
Politechnika Białostocka
Wydział Budownictwa i Inżynierii Środowiska, Zakład Informacji Przestrzennej
ul. Wiejska 45E 15-351 Białystok
tel. 85 746 96 92/ fax: 85 746 95 59, e-mail: e.kozniewski@pb.edu.pl; m.orlowski@pb.edu.pl

GEOMETRICAL ASPECTS OF AN INSOLATION OF A MULTI-SIDE ROOF'S SURFACE

The main aim of this paper is formulation of assumptions and research methods of influence of detached house multi-side roof's geometry on amount of solar energy consumption, taking into consideration the roof's location in respect to the four quarters of the globe, within a range of direct radiation.

As a starting point to deliberation we take an empirical formula of solar radiation components: direct ray I_b, diffused ray I_d and reflected ray I_h:

$$I = I_b \cdot \cos\theta + \frac{I_d\,(1+\cos\beta)}{2} + \frac{I_h\,(1+\cos\beta)}{2}\ ,\ [W/m^2] \tag{1}$$

where: θ stands for an angle enclosed between two lines: one that is appointing the direction of a direct solar radiation and the other normal to the absorber's surface (see fig.1a), β is an angle of collector's surface inclination in relation to level. In a formula (1) component of direct radiation depends on the solar rays' angle of incidence $-\theta$. We may formulate *wnb* index which describes a dimension of insolation:

$$wnb = \Sigma\, \boldsymbol{P}_i \cdot \cos\theta_i\ [m^2], \tag{2}$$

where $\boldsymbol{P}_i$ stands for sunlit surface area $\mathbf{P}_i$, and θ_i is the solar rays' angle of incidence on the surface $\mathbf{P}_i$, (i=1,2,3,4).

The problem of description of solar energy dimension absorbed by a given surface in the course of a certain period of time (*wnbt* index will be formulated with an appropriate integral after taking into consideration the time) is very complex. It requires a formulation of a general algorithm of shade evaluation or to be more precise solar rays' illumination for a specific class of geometrical surfaces. We will restrict our deliberation to chosen multi-side surfaces (roofs) and in selected days of the year. Established method remains quite universal and indicates research directions in order to find a solution for a general issue.

In the mathematical description of this phenomenon we will make use of *spherical system,* where the centre is in an observation post, main circle - a horizon and main direction - geographical direction of South (S).We assume that the Earth in an observation post, is flat, and the horizon seen from that point is motionless and describes a perfect circle. Then, solar coordinates are: *solar azimuth* α defined as an angle between equator's semi-surface at the southern side and a semi-surface of a vertical circle crossing the Sun, and *astronomical altitude* ε that is an angle between horizontal surface and the direction of solar rays' incidence. For their description *declination* δ, that is an angle between vector connecting the Earth with the Sun and the equator's surface is needed. That declination can be calculated from Cooper's formula

$$\delta = 23{,}45^{o}\sin(360^{o}\,\frac{284+n}{365}), \tag{3}$$

where n stands for the number of the day within a year.

Subsequently we appoint so called "hourly Sun's angle" counted as:

$$\omega = 15^{o}\,(12 - t_s), \tag{4}$$

where t_s stands for the time counted in hours. At 12 p.m. the dimension of an angle ω comes to 0^{o}. An hourly angle changes with a speed of 15 degrees per hour.

For Bialystok, which latitude φ comes to $53^{o}08'$, we can distinguish the following time intervals of the Sun operating on the horizon: 21.03–(6:28,18:42); 22.06–(3:59, 20:59); 23.09–(6:13,18:25); 22.12–(8:38,16:43).

Taking as a stroke 20 minutes we create a sequence of angles from the formula:

$$\varepsilon = \arcsin(\sin\varphi\sin\delta + \cos\varphi\cos\delta\cos\omega) \tag{5}$$

with a stroke of 5^{o} as a base for a table of elevation angles.

We appoint azimuth angle α from the formula:

$$\alpha = \arccos\left(\frac{\sin\varepsilon\sin\varphi - \sin\delta}{\cos\varepsilon\cos\varphi}\right). \tag{6}$$

An exemplary shade calculation of roof slopes for two variants of roof's arrangements looks as follows:

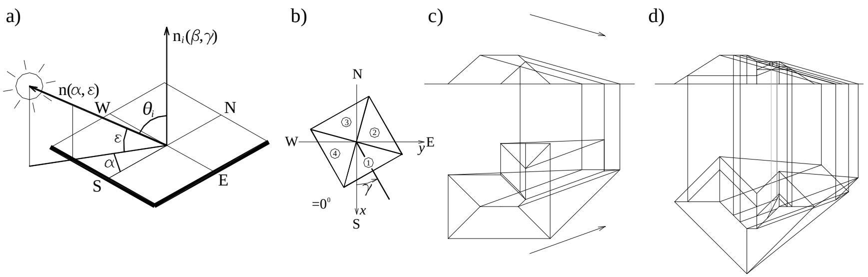

Fig.1

After appointing the shades, reading of an insolated surface takes place (fig. 1c, d). In order to appoint the solar rays' incidence angle on any roof slope and above any right-angled polygon we can take as a model a pavilion roof with a slope β. We appoint the position of a building through the angle γ related to the selected

roof slope. As we can see on the figure 1b, the selected roof slope, which defines the position, is roof slope 1. All roof slopes have exactly four (i=1,2,3,4) different locations in respect to the four cardinal points. According to those assumptions we may estimate the unitary vector of solar ray $\mathbf{n}(\alpha,\varepsilon)$ as a

$\mathbf{n}(\alpha,\varepsilon)$ = [cosα cosε, sinα cosε, sinε]. The unitary vector $\mathbf{n}_i(\beta,\gamma)$ which is perpendicular to the roof slope i, (i=1,2,3,4) has coordinates $\mathbf{n}_i(\beta,\gamma)$ = [sinβ cos(γ+(i-1)90^o), sinβ sin(γ+(i-1)90^o), cosβ]. Therefore the angle's cosines equals scalar product $\theta_i = \mathbf{n}(\alpha,\varepsilon)\circ\mathbf{n}_i(\beta,\gamma)$ or

$$\cos\theta_i = \cos\varepsilon\,\sin\beta\,(\cos\alpha\cos(\gamma+(i-1)90^o) + \sin\alpha\sin(\gamma+(i-1)90^o)) + \sin\varepsilon\,\cos\beta. \qquad (7)$$

Conducted calculations demonstrate that the building from the figure 1c has higher index of roof's insolation.

Literature:

[1] Wiśniewski G., Gołębiewski S., Gryciuk M., Kurowski K.: *Kolektory słoneczne. Poradnik wykorzystania energii słonecznej*. Wyd. III. Centralny Ośrodek Informacji Budownictwa, Warszawa 2006.