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GENERALISED PAPPUS THEOREM

The well-known theorem of Pappus, as one of the most important theorems of the projective geometry, was a subject of many investigations. We may mention here for instance the work [1], where this theorem was generalised to the n-dimensional projective space P^n (projective space over an arbitrary commutative field). This generalisation concerns two sets of points $A=\{a_0,...,a_n\}$and $B=\{b_0,...,b_n\}$ on two hyperplanes H_1 and H_2, respectively. The theorem says that the dimension of the join of subspaces (points in general) $S_0,...,S_n$ is not greater than n-1 ($S_j = \bigcap_{i=0, i\neq j}^{n} S_{ij}$, where $S_{ij} =J(b_i, A\backslash\{a_i,a_j\})$, $i\neq j$ (the symbol $J(P_1,...,P_m)$ denotes the join of subspaces $P_1,...,P_m$). Points $a_0,...,a_n$ as well as $b_0,...,b_n$ are assumed to be in a general position i.e. no n of them are in an (n-2)-dimensional subspace. Obviously, when n=2, it is the usual plane Pappus' theorem.

In this work we present a more general theorem than that from [1]. Throughout the paper we investigate two sets of points $A=\{a_0,...,a_n\}$and $B=\{b_0,...,b_n\}$such that $\dim J(A)=n-1$, $\dim J(B)=k$, $1\leq k\leq n-1$, and points $a_0,...,a_n$ as well as $b_0,...,b_n$ are in a general position (no k+1 points of $b_0,...,b_n$ are in a (k-1)-dimensional subspace). Under the above assumptions we prove the following

Theorem.

If $\dim J(A)=n-1$ and $\dim J(B)=k$, $1\leq k\leq n-1$ and J(B) is not included in J(A), then $\dim J(S_0,...,S_n)\leq k$.

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