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CONSTRUCTIVE – ANALYTICAL REPRESENTATION OF RULED MANIFOLD OF MULTIDIMENSIONAL SPACE.

It is necessary to develop algorithm of a constructive-analytical representation of the model for definition of this model or the approximating/interpolating equation of the technical shape or multifactorial process of a multicomponent system. Thereupon, the decision of a given problem in this paper is considered by example structurally - analytical representation ruled hypersurfaces of four-dimensional space.

One-parametrical variety of two planes forms a hypersurface.

As hypersurface of four-dimensional space is called the three-parametrical manifold 0 – the planes, satisfying to the certain law, or two-parametrical manifold 1 – planes or 1 – surfaces, and at last, one-parametrical variety 2 – planes or 2 – surfaces.

Let's admit, that as the forming it is set 2 – a plane. It is easy to notice on the basis of formula Grassman, that in four-dimensional space E_4 six-parametrical variety 2 – planes.

The Calculation Formula of a dimension of the Grassman manifold define as [1]:

$$D_n^m = (m+1)\cdot(n-m),$$

where m – dimension of a ruled object,

n – dimension of a space.

$$D_n^m = (2+1)\cdot(4-2) = 6.$$

This condition in symbols of enumerative geometry [2] can be represented as $e^{210}_{431}=1$

It means, that crossing 1 – surfaces and 2 – planes, and to its dimension is equal 0.

Dimension of this condition is defined from the formula:

$$Q = \frac{(2n-m)\cdot(m+1)}{2} - \sum_{i=0}^{m} a_i$$

where a_i - dimension of planes.

$$Q=\frac{(2\cdot 4-2)\cdot(2+1)}{2}-(3+5)=1$$

In the symbolical form the hypersurface of the four-dimensional space can be presented as $e^{210}{}_{430}\cdot(e^{210}{}_{431})^3$, after a transforming we shall get, that $e^{210}{}_{430}\cdot(e^{210}{}_{431})^3=2\cdot e^{210}{}_{310}$. After multiplication $2\cdot e^{210}{}_{310}$ on $e^{210}{}_{431}$ we shall receive $2\cdot e^{210}{}_{210}$. That it's a second-order hypersurface.

The theorem: If ruled the hypersurface is set by a point and three in pairs crossed straight lines the order of such hypersurface will be equal to two.

Consequence: If two straight lines are crossed, the hypersurface breaks up on two hyperplanes (figure1).

Let's present such hypersurface on drawing Radisheva. We set a point A and three in pairs crossed straight lines a, b, c.

Let's specify algorithm of formalized graphic-analytical representation.

1. To choose forming hypersurfaces and to define dimension its variety in space E_4.
2. To make up a set of geometrical conditions of incidence parallelism and perpendicularity for a given forming and to define their dimensions.
3. On the basis of dimensionality conditions, to pick up by formal attributes their number satisfying with the definition of a hypersurface.
4. To verify the chosen conditions on compatibility and to define an order of a designed hypersurface.
5. To present the chosen conditions in an analytical kind.
6. To derive the analytical equation for the singled hypersurface.

To deduce the analytical equation for the chosen hypersurface (figure2).

We choose on a straight line and a point 1, it is any way connected it to a point A, to find one of the two-dimensional planes, forming this hypersurface. We build missing projections of the chosen surface. We find crossing of this straight line with a hyperplane which is set by straight lines b and c, we receive B. Let's find a straight line which will pass through a point B and to cross b and c about one of two-dimensional planes who is forming.

Example: we will construct the hypersurface of the second order set by a point and three in pairs crossed straight lines, in space E_4. Initial data will be 7 points from construction and any 7 points.

_1x + _2y + _3z + _4t + _5xy + _6xz + _7xt +_8yz + _9yt + _
10zt + _11x^2 + _12y^2 + _13z^2 + _14t^2 -1 = 0.

We receive 14 homogeneously – the linear equations, solving them by means of mathematical package Maple.

```
> with(LinearAlgebra):
> points :=
```

Mtrix([[30,27,110,27],[218,108,40,124],[256,62,85,80], [100,56,60,47],[204,41,54,52], [164,85,93,118], [192,71,97,111],[68,59,63,46],[76,46,93,50],[157,76,69,92],[246,45,104,96], ,[225,38,56,50], ,[185,55,60,68],[108,113,85,134]]);

```
> getmatrix := proc(X)
> if not type(X, Matrix)
then error "No Matrix!"; end if:
m := Matrix(RowDimension(X),14):
for i from 1 to RowDimension(X) do
  m[i,1] := X[i,1]:
  m[i,2] := X[i,2]:
  m[i,3] := X[i,3]:
  m[i,4] := X[i,4]:
  m[i,5] := X[i,1]*X[i,2]:
  m[i,6] := X[i,1]*X[i,3]:
  m[i,7] := X[i,1]*X[i,4]:
  m[i,8] := X[i,2]*X[i,3]:
  m[i,9] := X[i,2]*X[i,4]:
  m[i,10] := X[i,3]*X[i,4]:
  m[i,11] := X[i,1]*X[i,1]:
  m[i,12] := X[i,2]*X[i,2]:
  m[i,13] := X[i,3]*X[i,3]:
  m[i,14] := X[i,4]*X[i,4]:
end do;
return m;
> end proc:
matr := getmatrix(points);
vectr := Vector(RowDimension(matr),-1);
X := LinearSolve(matr, vectr);
```

We receive factors of the equation of a hypersurface.

-0.01160880639
0.0009720795744
0.03581376936
-0.02924924596
0.00007865479714
-0.00002316776932
0.000005706375152
-0.0007164649816
0.0008769958717
0.0007469019731
0.00002051130830
-0.0003025952593
-0.0003300635086
-0.0004848744720

Conclusion: In this paper the algorithm of formalized grapho-analytical presentation of hypersurfaces in the space E4 by means of methods of the parameterization theory and enumerative geometry is offered. That allows to establish a kind of approximating/interpolating equation and to define number of derivation.

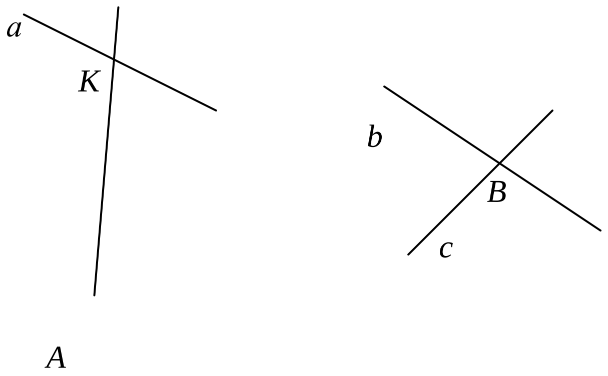


Fig. 1 Scheme of algorithm of a special case of an arrangement of initial data.

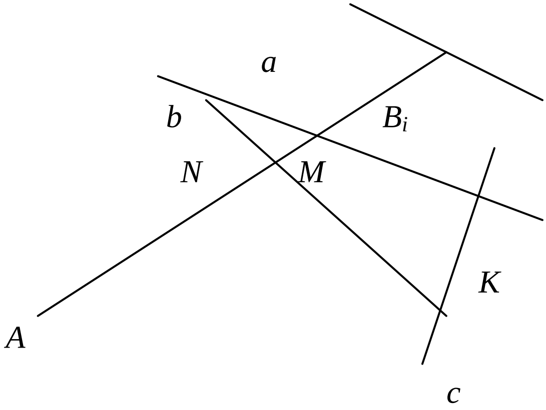
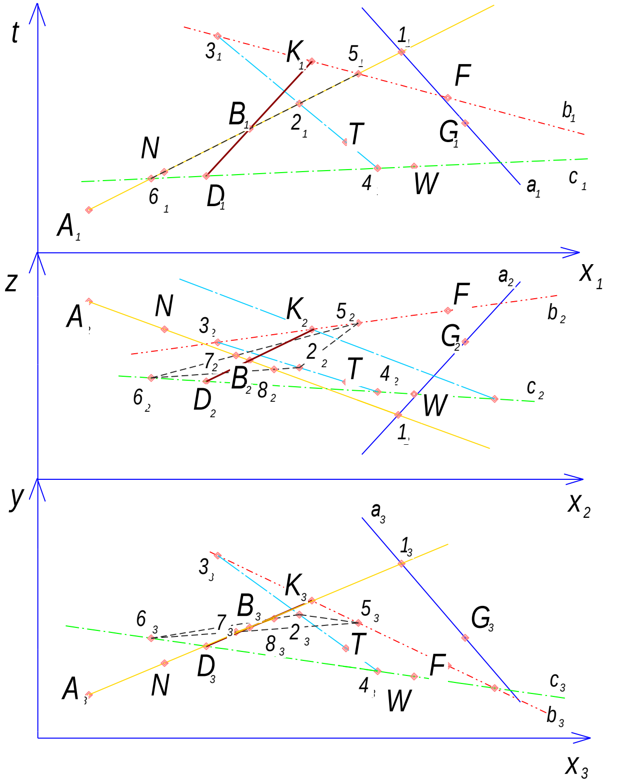


Fig. 2 Scheme of algorithm of construction of one of two-dimensional 2 – planes forming one of hypersurfaces.



The hypersurface of the second order set by a point and three in pairs crossed straight lines

Fig. 3 The hypersurfaces of the second order set by point and three in pairs crossed straight lines.

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