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MINKOWSKI SUM IN GEOMETRIC MODELLING

The main idea of this paper is to present some of the basic facts on Minkowski sum of point sets and to show several examples illustrating calculation of the sum of convex sets, e.g. n-gons. Several applications of Minkowski sum in modelling of zonotops are included, and generation of surface patches in the Euclidean space as Minkowski sums of differentiable manifolds defined by their vector representations is described.

Minkowski sum of two point sets is a binary geometric operation defined on point subset of the Euclidean space, and it can be interpreted in different ways. Operation was defined in 1903 by Hermann Minkowski. The most natural way of the Minkowski sum interpretation is using the vector sum of the positioning vectors of all points in the respective operands – point sets in the Euclidean space. Next frequently appearing interpretation is a continuous movement of one operand on the boundary of the other one without the change of orientation. Applications of Minkowski sum can be found in computer graphics and robotics, in searching for algorithms of dense placement and layout and offsetting, in geometric modelling, in CAD, and in many other fields.

Let A and B be two point sets in the n-dimensional Euclidean space E^n.

Definition 1. Minkowski sum of sets A and B in the Euclidean space is such set, which is the sum of all points in set A with all points in set B, i.e. set

$$A \oplus B = \{a + b; a \in A, b \in B\}. \tag{1}$$

Definition 2. Minkowski sum of sets A and B in the Euclidean space is set

$$A \oplus B = \bigcup_{b \in B} A^b , \tag{2}$$

where A^b is set A translated by vector b

$$A^b = \{a + b; a \in A\}. \tag{3}$$

Minkowski sum of two line segments is a parallelogram or a line segment, if both operands belong to the same space E^1. Minkowski sum of 3 non-coplanar line segments is a parallelepiped, and Minkowski sum of n line segments not located in one hyperplane of the n-dimensional space is a zonotop, or zonohedron with all antipodal edges parallel and equal. Zonotop facets are zonotops itself, but of a lower dimension. Some of special zonotops are illustrated in Fig. 1.

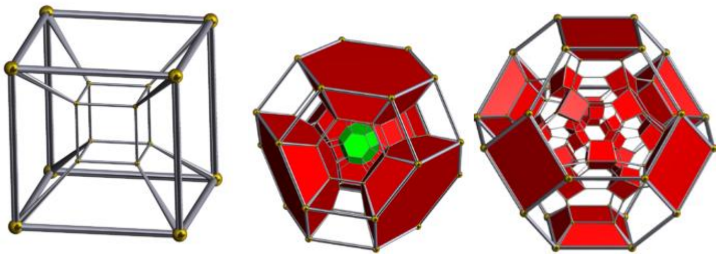

Fig. 1 Tessaract (left), truncated 5-cell (middle), truncated 24-cell (right).

Easy examples of the Minkowski sum of 2 curve segments are surface patches in Fig 2.

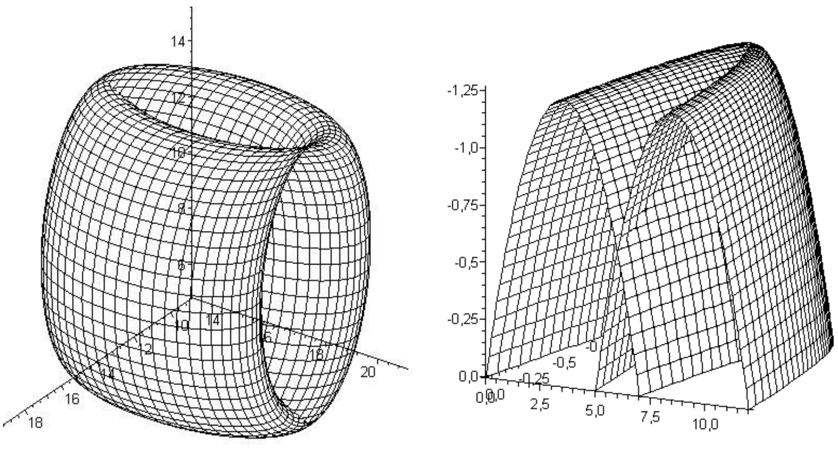

Fig. 2 Minkowski sum of 2 ellipses (left), and 2 parabolic arcs (right).