Daniela VELICHOVÁ
Slovak Technical University
Department of Mathematics, Mechanical Engineering Faculty
Námestie slobody 17, 812 31 Bratislava, SLOVAKIA
Tel.: +4212 5729 6115, fax: +4212 5292 5749, e-mail: daniela.velichova@stuba.sk

GEOMETRIC MODELLING OF KNOTTED TORI.

In the presented paper, the knotted torus will be understood as a regular surface created by a knotting of a torus like surface (not solid) in the three-dimensional space $\mathbf{E}^3$. Knotted torus is a surface with no self-intersections and singular points, it is closed, and its envelope is again a torus. This means that by knotting a torus no new type of topological structure will be created, or no new type of knot will arise than the original one. Consequently, there exist infinitely many different forms of the knotted tori in the three dimension space.

From the geometric point of view we can assume that a knotted torus is an envelope surface created by the continuous movement of the sphere in the space with the trajectory located on the torus of revolution in such way that no self-intersecting might occur. This curve is in the general case trajectory of the specific composite revolutionary movement about the two skew axes (interior axis ${}^1o$ and exterior axis ${}^2o$) called Euler movement known as the Euler trajectory and described in details in [1]. Simple Euler trajectory in the basic form is a closed space curve without any multiple points; its orthographic view in the plane perpendicular to the axis of revolution ${}^2o$ is a symmetric plane curve, Limaçon of Pascal. Euler trajectory is located on torus of revolution with the axis in the exterior axis of revolution ${}^2o$, and radius equal to the distance a of the moving point from the interior axis of revolution ${}^1o$. Coordinates of the Euler trajectory points satisfy the implicit equation of the torus in the form

$$\left(x^2 + (y-d)^2 + z^2 + d^2 - a^2\right)^2 = 4d^2\left((y-d)^2 + z^2\right).$$

There can be distinguished 4 different types of knotted tori, with respect to the position of the basic circle and the two axes of revolutions. Shaping parameters – angular velocities of the two simultaneous revolutions define the specific forms of these surfaces

Knotted tori of the first type are surfaces in the group of cyclical toroidal two-axial surfaces of revolution of Euler type (complete classification is presented in [2]), which can be generated by the movement of the basic circle g located in the plane passing through the interior axis of revolution ${}^1o$. These consist of several threads and knots. Knotted tori of the second type are modelled as cyclical normal two-axial surfaces of revolution of Euler type. Basic circle of these knotted tori is located in the plane passing through the exterior axis of revolution ${}^2o$. Surfaces consist of multiple knots with more threads. Knotted tori of the third type are surfaces in the group of cyclical general two-axial surfaces of revolution of Euler type; these can be generated by the movement of the basic circle g located in the plane perpendicular to the interior axis of revolution ${}^1o$ and not passing through the exterior axis ${}^2o$. Knotted tori of the last fourth type are modelled as cyclical general two-axial surfaces of revolution of Euler type with basic circle located in the plane perpendicular to the exterior axis of revolution ${}^2o$.and not passing through the interior axis ${}^1o$.

Some illustrations of the knotted tori forms are presented in the fig. 1.

Fig. 1 Forms of knotted tori

References

[1] D. Velichová: *Trajectories of composite rotational movements*, G – Slovak Journal for Geometry and Graphics, ISSN 1335- X, Vol. 3, No. 5, 2006, pp. 47-64.

[2] D. Velichová: *Classification of two-axial surfaces*, G – Slovak Journal of Geometry and Graphics, ISSN 1335- X, Vol. 4, No. 7, 2007, pp. 63-82.

[3] N. D. Gilbert, T. Porter**:** *Knots and Surfaces*, Oxford University Press, 1997 (original 1994).

[4] J. C. Turner, P. Van De Griend**:** *History and Science of Knots*, World Scientific Publishing Co., 1996.

[5] http://www.earlham.edu/~peters/knotlink.htm.