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TYPES OF GRAPHICAL SUBSPACE PROJECTINGS

The application of a so-called **graphical subspace projecting** is one of the most popular ways to arrive at the graphical representations of geometrical relations defined in the $\boldsymbol{P}_n$, $\boldsymbol{A}_n$ or $\boldsymbol{E}_{An}$ spaces. The realization of this type of projecting, described by the **RS** symbol, requires creation in the mapped $\boldsymbol{P}$ ($\boldsymbol{P}_n$, $\boldsymbol{A}_n$ or $\boldsymbol{E}_{An}$) space of a so-called **apparatus of projecting**, which consists of:

a) the continuous set $\{\mathcal{S}\}$ of equal-dimension subspaces $\mathcal{S}_i$ with dimensions (**n**-2) or (**n**-3), referred to as **centers of RS projecting**, which are elements of an established bundle of subspaces or subspace generators of an established second-degree form,

b) the projective relation **Z** that attributes centres $\mathcal{S}_i \in \{\mathcal{S}\}$ of the projecting **RS** to individual X_i points of representing $\boldsymbol{P}$ space, more precisely, the relation **Z** is converting (X_i O $\mathcal{K}$) elements of the established bundle $\langle \mathcal{K}, \boldsymbol{P} \rangle$ into the $\{\mathcal{S}\}$set; $X_i \Rightarrow \mathcal{S}_i = \mathbf{Z}$ (X_i O $\mathcal{K}$), **c)** the junction relation that establishes projecting form $\mathcal{R}_{Xi}$ = (X_i O $\mathcal{S}_i$) for every X_i point of $\boldsymbol{P}$ space, where $\mathcal{S}_i$ is the centre of projecting **RS** attributed to point X_i,

d) the Π form of projection that is a plane $\boldsymbol{\pi}$ or an appropriately selected second-degree $\hat{\boldsymbol{\tau}}$ surface; **RS** projecting is transforming the represented $\boldsymbol{P}$ space onto the Π form of projection, whereby in this projecting of the $X_i^{\mathcal{S}}$ image – projection of the X_i point is identical with the $\Pi \cap \mathcal{R}_{Xi}$ meet.

One can point out many types of projectings in the family **RS** graphical subspace projectings that are applicable in current or newly created reversible methods of graphical representation of multidimensional projective, affinic or Euclidean spaces. The main types among them include:

a) the RC subspace bundle projecting into which apparatus the $\{\mathcal{S}\}$ set of projection centres is reduced to an isolated $\mathcal{S}$ subspace, which is generally situated towards the

π plane of projection and has a dimension of (**n**-3) in case of an ordinary projecting (dim($\mathcal{S} \cap \pi$) = -1), or of (**n**-2) in case of a generalized projecting (dim($\mathcal{S} \cap \pi$) = 0),

b) the RB subspace projecting with bundle dispersed centres characterized by the fact that centres $\mathcal{S}_i$ of projecting are assigned to individual X_i points of projected the ***P*** space as subspaces $\mathcal{S}_i$ about dimensions (**n**-2) why (**n**-3) and being elements of the definite bundle $\langle \mathcal{C},\mathcal{F} \rangle$. The attribution is conducted through a correct projective relation **Z** between bundles $\langle \mathcal{C},\mathcal{F} \rangle$ and $\langle \mathcal{K},\boldsymbol{P} \rangle$ having the same factors of manifolds; $\mathcal{S}_i = \mathbf{Z}(\mathcal{L}_i(X_i,\mathcal{K}))$,

c) the RQ subspace projecting with centres dispersed on a second-degree form; in which case in order to define an apparatus of projecting we establish in the ***P*** mapping space three bundles $\langle \mathcal{K},\boldsymbol{P} \rangle$, $\langle \mathcal{C}_1, \mathcal{F}_1 \rangle$ and $\langle \mathcal{C}_2, \mathcal{F}_2 \rangle$ with identical factors of manifolds. These bundles have the following properties: **1º** dim $\mathcal{F}_1$ = dim $\mathcal{F}_2$, dim $\mathcal{C}_1$ = dim $\mathcal{C}_2 \in \{(\mathbf{n}\text{-}3),(\mathbf{n}\text{-}2)\}$, **2º** $\mathcal{F}_1 = \mathcal{F}_2$ and $\mathcal{C}_1 \neq \mathcal{C}_2$, when defining an apparatus for the **RQ** projecting that belongs to the family of so-called **meet projectings**, **3º** $\mathcal{F}_1 \neq \mathcal{F}_2$ and $\mathcal{C}_1 = \mathcal{C}_2$, when defining an apparatus for the **RQ** projecting that belongs to the family of so-called **junction projectings** . Furthermore, we are establishing the **Cl** colineation and the **Cr** correlation that transform the bundle $\langle \mathcal{K},\boldsymbol{P} \rangle$ appropriately onto the $\langle \mathcal{C}_1, \mathcal{F}_1 \rangle$ and $\langle \mathcal{C}_2, \mathcal{F}_2 \rangle$ bundles. The established bundles and defined projective transformations allow us to assign to every point $\boldsymbol{X}_i \in \boldsymbol{P}$ the following three subspaces: **1º** $\mathcal{L}_i = \boldsymbol{X}_i \,\mathrm{O}\, \mathcal{K}$, **2º** ${}_1\mathcal{D}_{Xi} = \mathbf{Cl}\,(\mathcal{L}_i) \in \langle \mathcal{C}_1, \mathcal{F}_1 \rangle$, **3º** ${}_2\mathcal{D}_{Xi} = \mathbf{Cr}\,(\mathcal{L}_i) \in \langle \mathcal{C}_2, \mathcal{F}_2 \rangle$ **.** Finally, we assume that the $\mathcal{S}_i$ centres of the **RQ** projecting for individual points $\boldsymbol{X}_i \in \boldsymbol{P}$ are, in case of a so-called meet subspace projection, the meets ${}_1\mathcal{D}_{Xi} \cap {}_2\mathcal{D}_{Xi} = \mathcal{S}_i$ with the dimensions equal to dim $\mathcal{C}_1$ = dim $\mathcal{C}_2$**,** and in case of a so-called junction subspace projecting, the junctions ${}_1\mathcal{D}_{Xi} \,\mathrm{O}\, {}_2\mathcal{D}_{Xi} = \mathcal{S}_i$ with the dimension equal to dim $\mathcal{F}_1$ = dim $\mathcal{F}_2$**.** In both cases, the centres $\mathcal{S}_i$ are subspace formers of an appropriate second-degree form**.**

Each of the described types of projectings **RC, RB** and **RQ** is being adapted to construction of panoramic images of three-dimensional spaces. Such an adaptation is based on accepting to the form of projection – a background of each of projectings the second degree bundle surface, whereas for centres of the projectings – an isolated point, a set of points of the straight line, or a set of points of a conic, which in each case belong to the inside of the background .