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THE PANTOGRAPH AND ITS GEOMETRIC TRANSFORMATIONS -A FORMER POPULAR TOOL FOR COPYING AND SCALING

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Abstract: Development of digital technology has resulted in significant transformations of designer workshops. Computer applications replaced the previously existing tools and methods for drawing up of projects. Professional, sophisticated drawing instruments and tools for manual plotting, as well as basic drawing aids, have gradually fallen into oblivion and become relicts of a bygone era - the objects of collectors' interest. Gdansk University of Technology prides itself on the possession of a great collection of old geometric tools, which serves not only to protect and promote the memory of past inventions that were used to support and improve the design processes, but also to inspire and develop creative imagination. The article describes the pantograph, considered to be the ancestor of today's photocopier, as it allows one to draw the copies and, at the same time, to enlarge or to reduce any figure. The author presents its genesis and the scope of application in the past. The construction of this tool and its geometrical transformations, which form the base of its action, are also explained.

Keywords: applied geometry, history of technology, geometric constructions

1 Introduction

The digital revolution of the designer workshop has happened recently before our eyes. Freehand and mechanical ways of design drafting have been almost entirely supplanted by computer tools. Until recently professional, sophisticated drawing instruments and tools for plotting, as well as basic drawing aids, have gradually fallen into oblivion and become relicts of a bygone era - the objects of collectors' interest. Gdansk University of Technology prides itself on the possession of a significant collection of old geometric tools among its historical resources. Such a rich and complete collection is unique in Poland. The exhibits from the collection of the late professor Henry Limon - a former student of the Faculty of Civil Engineering and employee of the Department of Descriptive Geometry - constitute its main core.

Professor Henry Limon had eventually left Poland and gone to live and work in Switzerland. His great passion was to collect old historic geometric drawing tools and engineers' instruments. After his death in 2009, his daughter Jowanka Saby from Geneva, according to the will of her father, donated a part of the collection to Gdansk University of Technology. The official ceremony took place in Gdansk in May 2014. The occasional exhibition of donated items took place at the same time. The ceremony participants had a chance to admire 50 exhibits. The collection includes both: basic drawing aids like rulers, triangles, protractors, various types of compasses and calipers, sets of styles (drawing pens), T-squares, rollers (used to draw parallel lines), and more sophisticated instruments such as ellipsographs (for plotting ellipses), pantographs (for copying and scaling), perspectographs (for plotting perspective views), and planimeters - instruments for calculating surfaces on maps. From this event on, a permanent exhibition of these drawing instruments has been on view at the university library and during particular exhibitions on the occasion of various

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events at the Faculty of Architecture. The collection serves to protect and promote the memory of past inventions that were used to support and improve the design processes. Young people are able to learn how design drawings were performed several decades ago, which can also inspire and develop creative imagination.

Three pantographs belong to the more interesting exhibits of the donated collection. These include: the brass pantograph made by Adrien Cavard Company and signed with the inscription 'Paris', the nickel-plated pantograph produced by Pierro Company (1948), and the large Russian tool made of steel from 1947.

The pantograph is worthy of deeper attention due to its popularity and wide range of applications in the past. It was a very useful, common and widespread equipment. Although nowadays it has fallen into disuse, the principle of its operation are applied in many modern devices.

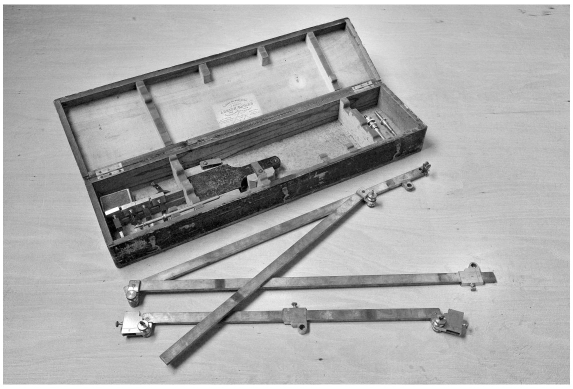

Illustration 1: Collection of Prof. H. Limon: Pantograph, company Adrien Cavard, Paris – photo K. Krzempek

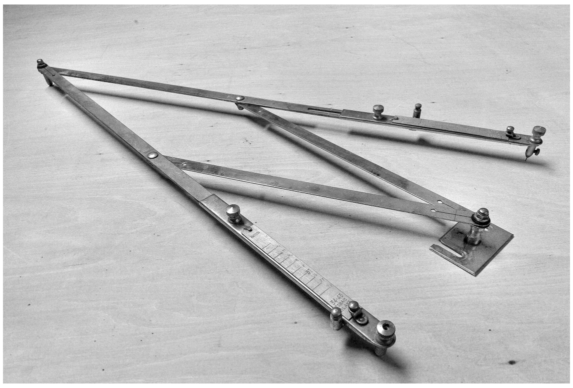

Illustration 2: Collection of Prof. H. Limon: Nickel-platedpantograph, company Cav. Pierro, Brevetto Fumagalli – photo K. Krzempek

2 Genesis of the pantograph

Since principles of pantograph operation are based upon Euclidean geometry, it is possible, that they were known and used from ancient times. The origin of this device derives from the Roman state. We know that Leonardo da Vinci (1452-1519) was using a pantograph to enlarge his drafts and transfer them onto canvas. Jesuit Christoph Scheiner (1575-1650), German mathematician and astronomer, is considered to be creator of the modern type of pantograph. In 1603 he built the first pantograph, which was able to reproduce drawings at different scales. He used it for copying and zooming diagrams - the results of his scientific work. This pantograph was described and illustrated for the first time in his work entitled 'Pantthographicae' (Rome, 1631). In 1831, in Edinburgh, William Wallace (1768-1843) developed a more sophisticated version of the pantograph, called eidograph.

3 Construction and the principle of operation

Pantograph (Greek: παντ- "all, every" and γραφ- "to write" - 'one that can write everything') is considered to be the ancestor of today's photocopier. It is a device for semi-automatic copying, reducing or enlarging drawings. It operates on the basis of simple geometric relations. The device is easy to use, however, the precision of operation depends on the details of its construction. Basically, the pantograph structure consists of a movable rod, which is integrated into a parallelogram with modifiable arm lengths used to achieve different scale factors. The arms are connected in a special manner so that they can move easily in fixed relationships to each other. The operation principle of the pantograph is based on the parallelogram, which is fixed at only one vertex. Other connections between its sides are movable, so that the parallelogram can freely change the vertex angles. The sides of the parallelogram are moving in a plane parallel to the drawing plane, when the axis of vertex rotations and tracing tools are perpendicular to it.

A graver and a scriber are respectively fixed to the arms of the parallelogram at certain points. While the graver traces the original figure, it changes the position of the parallelogram. At the same time, the scriber draws a copy of it, for instance on a piece of paper. Depending on the location of the graver and the scriber we may obtain a high fidelity copy of the figure, whose size is held constant, reduced or enlarged.

The name 'pantograph' is used also to denote a flat articulated mechanism. It is widely used as a fixing method, which allows the user to change the length of an arm easily.

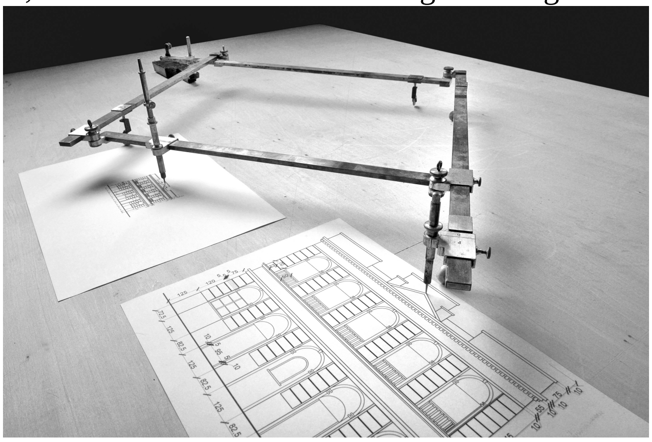

Illustration 3: Pantograph operation - photo Krzysztof Krzempek

4 Geometric transformations

Making copies of drawings by means of pantographs is based on dilation. Dilation with center O and scale factor $k \in$ R - {0} is the transformation of the plane to the plane, which assigns point A' to any point *A*. Point A' lies on the OA ray, and the distances between the points fulfill the condition:

$$|OA'| = k \cdot |OA|, \tag{1}$$

where the coefficient k is the ratio of dilation.
If $k = 1$ then dilation is an identity mapping,
If $k = -1$ then dilation is the central symmetry (we get a mirror image).
Each dilation is the similitude of the scale factor $|k|$, because two figures f &g are similar, when there is a point O and a non-zero scale factor k, such that the dilation converts the figure f to the figure g. If the coefficient $k > 0$, we deal with the dilation positive (points A and A' lie on the same side of the point O), but if $k < 0$, dilation is negative (points A and A' are on the opposite sides of the point O). The reverse transformation to a dilation with center O and scale k is the dilation with center O and scale $1/k$. A superposition of two dilation with center O and $k1$ and $k2$ scales is the dilation with center O and scale $k1 \cdot k2$.

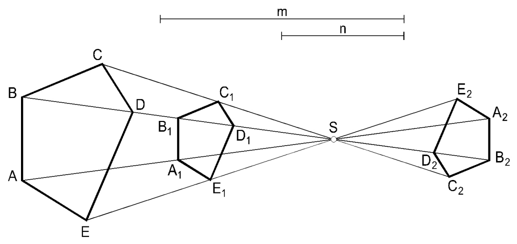

Illustration 4: Dilation of the pentagon ABCDE: for figure $A_1B_1C_1D_1E_1$: $k > 0$ & $k = n/m$, for figure $A_2B_2C_2D_2E_2$: $k < 0$ & $k = -n/m$

5 Various patterns of structures

Using the same geometric transformations, pantographs differed in the details of construction and modes of actions. Initially very simple schemas of geometric transformations were used. The design patterns of these tools have evolved towards obtaining the highest accuracy and flexibility in their work.

The diagram in Figure 1 refers to pantographs, which enabled significant enlarging or reducing of drawings. The line segments RA, BC, CD, AO form the sides of a parallelogram (Fig. 1). The connections at the points A, B, C, and D are movable, which allows one to change the vertex angles of the parallelogram. The axes of rotation at points A, B, C, and D are perpendicular to the plane of the drawing. Point O is the fixed point of attachment to the drawing plane.

If the points O, C and R, belonging to the parallelogram sides, lie on one straight line, and the pantograph has a constant size, and it is fixed at one of these three points, then during the movement of the parallelogram and changes of the position and size of its vertex angles, the other two points always circle similar figures.

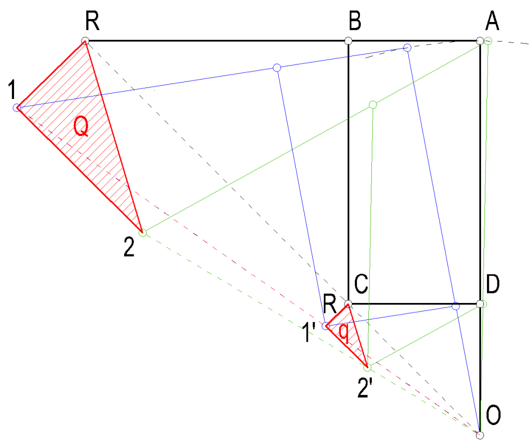

Figure 1: Point C, in which the copy is drafted, is the vertex of parallelogram

The ratio of dimensions of similar figures, which are plotted at the points R and C, is the same as the ratio of the lengths (absolute value) of the segments AR and AB:

$$\mathrm{q} \sim \mathrm{Q} \rightarrow k = |AR| : |AB|, \qquad (2)$$

where the symbol ‘q ~ Q’ denote the dilation of triangles q and Q (exactly: Q is the image of q in the dilation with the coefficient k and the center O). The size of figures drawn at point C can be changed by moving the position of O, C and R points, but so that they always lie on one straight line.

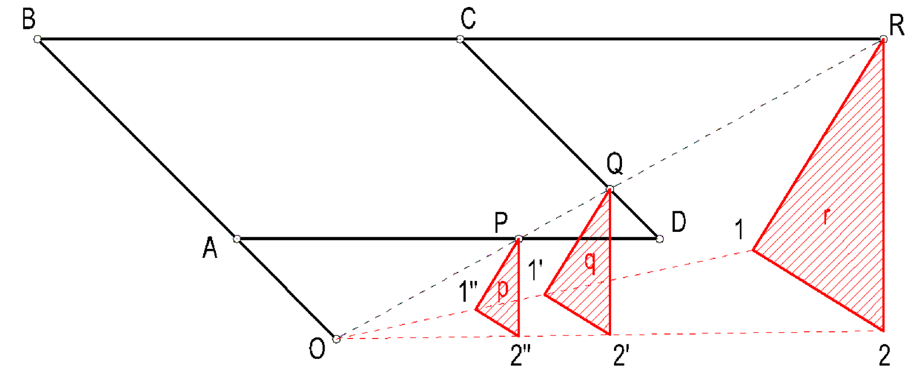

Figure 2: Points P & Q, in which the copies are drafted, are selected on the sides of the parallelogram

In Figure 2 the triangle r is transformed to the triangle q and to the triangle p, so:

$$\mathrm{r} \sim \mathrm{p} \to k = |AP| : |BR|, \tag{3}$$

$$\mathrm{r} \sim \mathrm{q} \to k = |BC| : |BR|. \tag{4}$$

In the case of transformations shown in Figures 1 and 2, fixed point O lies on the extension of the side of parallelogram and this is the center of transformation.

In the case of Figure 3, the attachment point of the pantograph and the center point of transformation are located at the vertex of the parallelogram

$$\mathrm{q} \sim \mathrm{r} \to k = |BR| : |BC|. \tag{5}$$

The pantographs' schematics, which are shown in Figures 1, 2 and 3, are used to enlarge or reduce pictures. Congruent copies are not generally possible for drawing.

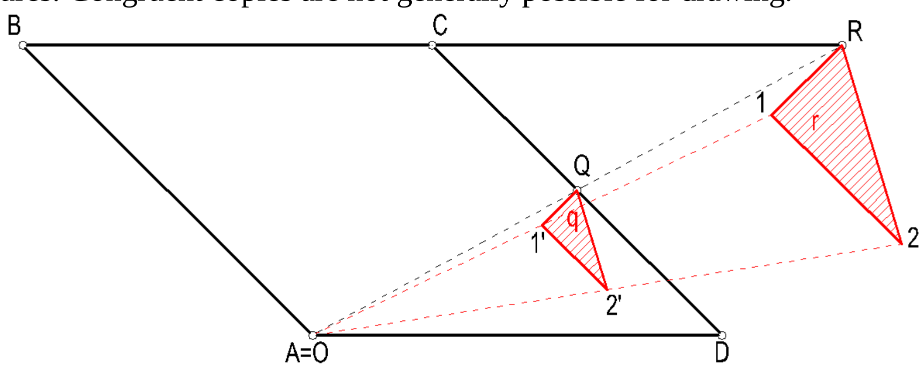

Figure 3: Fixed point O (the center of transformation) lies in the vertex of the parallelogram

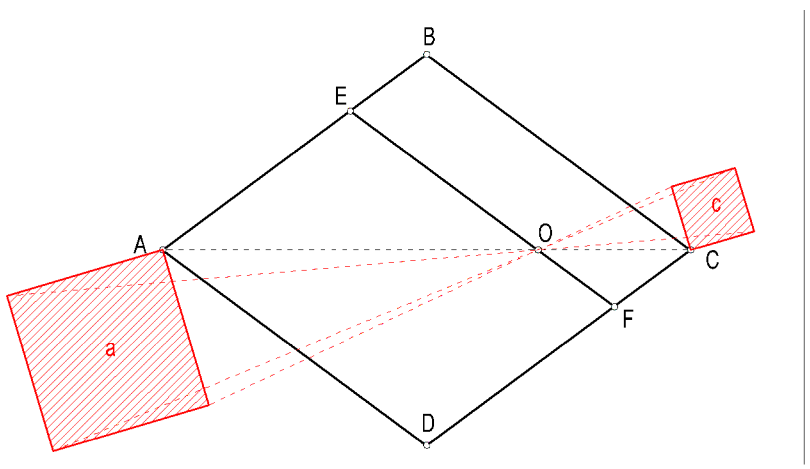

Figure 4: Pantograph in the shape of a parallelogram divided by a crossbar

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This type of pantograph (Fig. 4) is made up of a parallelogram ABCD divided by a crossbar, which is parallel to two sides of the parallelogram. Two from three collinear points, lie on opposite sides of the parallelogram, and point O lies on the crossbar. A stylus and a pencil are attached at the points A and C. The ratio of the dimensions of the triangles a and c, is equal to:

$$\mathsf{a} \sim \mathsf{c} \rightarrow k = |AE| : |EB| = |DF| : |FC| \qquad (6)$$

and has a negative value. The dilation is negative – the figures lie on the opposite sides of the point O – the center point of transformation. If the lengths AE = EB and DF = FC (the crossbar divides the parallelogram in half), then the dimensions of the figures a and c are equal and we obtain a mirror image. Central symmetry is the applied transformation, for which the coefficient of the scale $k = -1$. To obtain congruent figures, as the result of the transformation, it is necessary to repeat copying twice.

The ratio of the length of the sides |AE| : |EB| (|DF| : |FC|) can be changed by moving the crossbar EF parallel to the sides BC and AD, and the point O on the bar EF. The points A, O, and C have to be collinear. The O point at the crossbar EF has also to be shifted, so that the points A, O, C would be collinear. We need one more transformation if we want to receive the figures in the same position. According to literature this type of pantograph has found the widest application.

6 Application

The use of the pantograph was very early adopted by artists to reproduce drawings and copy them onto canvas. Sculptors and carvers also used the pantograph. The outlines of drawings were being entered onto blocks of marble or wood as guides for carving. Towards the end of the 18th century, pantographs had been used to typeset printed material - to cut letters in wooden blocks. They also proved to be indispensable in the engraving of gold and silver objects, even on surfaces that were not flat.

Subsequently sculptors sought to improve the pantograph, to make copying of 3Dspace possible. First they developed the pantograph to duplicate low-relief carvings. By the 19thcentury, the so-called swing-arm pantograph was used for duplicating complex elements. It was able to do it much faster and more accurately than by hand. Thanks to the swing-arm pantograph, there was an explosion of interest in reproductions of antique statues in the Victorian era. One of the most popular statues copied by this method was “David” by Michelangelo. Portrait busts of famous figures in literature, politics and music of those days also became popular and affordable.

At present the pantograph is still used in a variety of machines for the manufacture of similar movements.

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PANTOGRAF A PRZEKSZTAŁCENIA GEOMETRYCZNE – DAWNY POPULARNY PRZYRZĄD DO KOPIOWANIA I SKALOWANIA

Rozwój technologii cyfrowej spowodował znaczące zmiany w warsztacie współczesnego projektanta. Aplikacje komputerowe zastąpiły wcześniejsze odręczne i mechaniczne metody sporządzania projektów. Dawne profesjonalne przyrządy kreślarskie, narzędzia do kreślenia czy też podstawowe pomoce rysunkowe stopniowo odchodzą w zapomnienie i stają się reliktami minionej epoki - obiektami zainteresowania kolekcjonerów. Politechnika Gdańska szczyci się posiadaniem wspaniałej kolekcji starych narzędzi geometrycznych. Służy ona nie tylko ochronie pamięci o przeszłych wynalazkach, które wspomagały proces projektowania I wykreślenia projektu, ale także inspiruje i pobudza wyobraźnię. W artykule opisano pantograf, który jest uważany za przodka dzisiejszej kserokopiarki. Instrument ten służy do rysowania wiernych kopii, powiększania lub pomniejszania rysunku. Autorka przedstawia jego genezę, zakres jego zastosowania w przeszłości, wyjaśnia budowę i relacje geometryczne wykorzystane w jego działaniu. Na podstawie konkretnego przykładu praca przedstawia praktyczne zastosowanie przekształceń geometrycznych w wynalazkach dawnych narzędzi kreślarskich.