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ON SURFACE GEOMETRY INSPIRED BY NATURAL SYSTEMS IN CURRENT ARCHITECTURE

Anna NOWAK¹, Wiesław ROKICKI²

Warsaw University of Technology, Faculty of Architecture,
Department of Structural Design, Building Structure and Technical Infrastructure,
ul. Koszykowa 55, p. 214, 00-659 Warszawa, POLAND
¹e-mail:anna.patrycja.nowak@gmail.com ²e-mail: wrokicki@poczta.fm

Abstract. The contemporary architecture is increasingly inspired by the evolution of the biomimetic design. The architectural form is not only limited to aesthetics and designs found in nature, but it also embraced with natural forming principles, which enable the design of complex, optimized spatial structures in various respects. The trends in the development of the architectural design including the elements of bionics lead to the search for optimal ways which would eliminate unnecessary geometry. Digital tools play an important role in this process. The application of the algorithms and computer programs for modeling of three-dimensional space structures leads to the formation of specialized digital tools, on account of which it is possible to generate optimized irregular spatial forms [1]. Reproducing the construction of structures and their analyses of biological processes are possible by improving generative design methods, which allow to deepen the knowledge of shaping elements in the natural world [2]. The design based on the principles of natural formation of structures requires that the processes be understood and described with the use appropriate mathematical models. The use of mathematical models that attempt to describe forms from the world of nature, in particular morphogenetic patterns. Among the morphogenetic aspects described by mathematical models occurring in nature, the most noteworthy are the Voronoi diagrams, the Fibonacci sequence and homologous transformations, which are increasingly used as methods of discretization of space in shaping structural surface.

Keywords: bionic, structural surfaces, mathematical models

1 Introduction

Today, integrating the form and structure in modern shaping of architectural constructions is increasing. As a result, synergistic methods of architectural design are gaining momentum, enabling the creation of more efficient systems. Exploring the possibilities of shaping the optimal and sustainable structures led to a return to nature. Modern inspiration nature not only leads to the direct transfer of the shape, but also how to build forms. As a consequence, bionic engineering is beginning to play an increasingly important role. Bionics as an interdisciplinary field of science investigating the functioning of living organisms and its possible applications created models that mimic biological systems in technology. Because of the way the bionic modeling works, it is possible to distinguish a design carrying biological models into technology or a cooperation between biological and technological progresses. Shaping the bionic architecture is also a way of searching for building structures that reflect biological systems, which is the result of imitating systems and processes occurring in nature [7]. As a result, it is possible to design building forms synergetic with their structures, but also shaping the facade systems and building components in a bionic way. The shaping of an object can be based on morphogenetic processes that determine the structure and elements from the natural world and transfer their functions or processes occurring in living organisms.
2 Mathematical models describing the bionic structure

Describing the biological structures is possible using the appropriate mathematical models. Mathematical models as a finite set of symbols, and mathematical relationships with certain principles of handling them, has the interpretation relating to specific elements of the modeled fragment of reality. The physical interpretation of an abstract set of mathematical relationships of the symbol and leads to the creation of a mathematical model [3]. As a result, the created models based on designs from the natural world that can be used in the technique [4]. Studies in this field were carried out at the beginning of the 20th century. Pioneer in research into the relationship of mathematics and biology was D'Arcy Wentworth Thompson. Research on the description of the morphogenesis as a process in which patterns are shaped with the natural world and homology were described in the book "On Growth and Form". Thompson insisted that the forms found in nature formed under the influence of the laws of physics and the development of organisms is mathematically responding mechanical phenomena [11]. The relationship between the form and the size described as allometry. He also worked on plithotaxis, a relationship between spiral structures leaves in plants, which developed the Alan Turing in the publication "The Chemical Basis of Morphogenesis". Michael Winstock emphasize that the morphogenesis of is based on the operations on mathematical models [13]. The geometric convergence of forms found in nature, relieve the complexity of the process of morphogenesis, which can be used in architecture [4].

The development of digital tools resulted in a transfer of these patterns to urban planning as well. On the other hand, the architectural shaping takes on the characteristics of forming irregular spatial structure models. Thanks to the improvement of generative design methods, the reconstruction of structures and the analysis of biological processes become possible, allowing to deepen the knowledge of the technology of shaping elements from the natural world [2]. The use of mathematical models makes it possible to describe forms from the world of nature, in particular morphogenetic patterns, which are being increasingly used as a method of discretizing space in the shaping of structural surfaces.

Figure 4: Left: Gold spiral seen in world of nature, for example, in fern. Right: Voronoi diagrams visible in the structure of the wings of dragonflies. Source: copyright materials

Due to the search of harmony and proportion in architecture, the golden ratio deserves to be mentioned as an morphogenetic aspect occurring in nature. One can easily notice the fascination with Fibonacci’s sequence, which describes the structure of the natural world in the proportion system of ancient buildings, however, the confluence within the canons and proportions used in architecture was discovered as late as in the nineteenth century, proposed
by Alfred Zeising [9]. In the 20th century, Alan Turing analyzed morphogenesis in the plant world, and the arrangement of leaves which is equivalent to a series of Fibonacci numbers [12]. The Fibonacci sequence is one sequence, the first and the second term of which is equal to 1 and each consequent \( n \) term is the sum of terms preceding \( n-1 \) and \( n-2 \). The golden number and golden ratio are deemed as a perfect proportion of harmony and beauty. The golden ratio determines the division of a segment into two parts so that the ratio of the longer to the shorter is directly proportional to the ratio of the sum of the segment lengths to the length of the longer one. The golden number also appears in the course of the golden function.

Another example used in the architecture of mathematical models describing the phenomena in the natural world are catenary curves, visible in the structure of a spider's web. The shape of the catenary curve at optimum stress does not break the spider’s web. The catenary curve is a perfectly flexible chain, both uniform and non-stretchable, hanging freely between two supports in a uniform gravitational field. The catenary curve is described by a hyperbolic cosine. In 1691 Gottfried Wilhelm Leibniz, Christian Huygens and Johann Bernoulli provided an equation of the catenary curve, undermining Galileo's assertion that the shape of a freely hanging chain was parabolic.

The cellular automata maps the inspiration with pigmentation patterns commonly found in the shells of certain snails of seemingly chaotic geometries. The mechanism occurring in the patterns of the shells was explained in the 50s of the twentieth century by Alan Turing who based his findings on the diffusion of cells. The ability of a cell to produce activators (at low activation level), pigments (at medium activation level), and inhibitors (at high activation level) results in the formation of patterns on the shells snails depending on cell stimulation. In 1990 Przemysław Prusiewicz observed a similarity on some sea snail shell ornaments to the geometry obtained using cellular automata. Formally, the cellular automata are assumed to be an ordered trinity, consisting of a network of cells, of an acceptable set of conditions which may be present in the cell and the function (rules) defining the transition state, in which the cell has to be in \( t + 1 \) step of the simulation [8]. The basics of cellular automata have been identified in the 40s of the XX century by Stanislaw Ulam and John von Neumann. The basic properties of cellular automata are similar to living organisms, their versatility, completeness, difficulty in predicting results, patterns of behavior, self-replication and self-organization [8].

Figure 5: Left: Gold spiral constructed on the squares with sides equal to the next Fibonacci expression. Right: Voronoi diagrams as a dual graph Delaunay triangulation. Source: copyright materials
The Voronoi tessellation describes the self-organization system of visible biological structures, among others, the divisions on the dragonfly wings, turtle shells, honeycombs or a sea urchin skeleton. The Voronoi diagram is a graph consisting of cells created from centers (seeds), node edges and Voronoi nodes (a point equally distant from at least three seeds). The Voronoi diagram consists of sections and half-lines forming edges of Voronoi cells that are created for each of the diagram center. The Voronoi cell is defined for each nucleus as a set of points in the plane closest to a given center. The Voronoi diagram is, formally, defined for a given set of n points as a plane divided into n areas in such a way that every point in any cell is closer to a specified point of a set of n points than the remaining n-1 points. Voronoi cell in two-dimensional space are convex polygons and in three-dimensional space are convex polyhedra. To construct Voronoi diagrams a progressive algorithm can be used, as well as by means of Divide-and-Conquer paradigm or Fortune’s algorithm [5]. Voronoi diagrams allow you to create an optimal grid built out of node points, which caused widespread scientific interest in shaping urban planning and architecture. In modern urban planning and architecture, the Voronoi diagrams are used thanks to the development in the digital tools design process. Many versatile computer programs are widespread and their innovative exploration possibilities lead to the creation of interactive applications and algorithms for the optimal development of structures.

The principles and methods of discretization and self-organization of living organisms inspire architects and designers to search for the optimal structures. In this very search, among others, a polyhedral foam structure was analyzed, the laws of which were empirically determined and described in 1873 by Joseph Plateau. According to these laws, three walls meet at an angle of 120°, three walls form a frame called the Plateau’s border and the four Plateau’s borders that converge at an angle of 109.5°. The Plateau’s laws and the relationship of surface area to volume have both been proven by Jean Taylor in 1976.

In 1887, William Thomson, also known as Lord Kelvin published a work called "On the Division of Space with Minimum Partitional Area", in which he presented the solution to resolve the issue by filling the space with the use of elements of identical shape and assuming a minimum ratio of surface area to the volume. As a solution to this issue, he presented a tetrahedron with eight hexagonal and six square walls. This model was considered the most perfect, despite the lack of mathematical evidence. The Weaire-Phelan structure, developed in 1993 by Denis Weaire and Robert Phelan, served as a continuation to these deliberations. The module of this structure includes six 14-sided polyhedra with pentagonal and hexagonal walls and two dodecahedra with pentagonal walls.

Three-dimensional quasicrystals, discovered in 1984 by Dan Shechtman in the cooled aluminum-manganese alloy, show characteristic properties of two-dimensional aperiodic tessellation structures. The seemingly regular structure of quasicrystals is not formed by filling the space with identical elementary cells, but due to the aperiodic tessellation, which provides a 5-fold and higher symmetry axes, unusual for the structure of conventional construction crystals [10]. Quasicrystals atoms are arranged in a quasi-periodic, so "almost repeating itself", whereas the spatial structure of the building is formed (simply put) by different geometrical bodies such as: rhombohedra, dodecahedra, icosahedra, triacontahedra. M. Duneau described the icosahedra quasicrystal skeleton structure as a solution to the 1985 Penrose problem.

Some aperiodic tessellations show characteristics of fractal geometry, e.g. the Pinwheel tiling. The tessellation of the Pinwheel tiling forms by dividing a right triangle with side lengths of 1, 2 and $\sqrt{5}$ into five isometric copies through scaling and tilting of the individual elements, which in turn leads to the formation of a fractal image. The fractal
geometry is generated by algorithms. The Koch curve is one of the first fractals described using mathematical models. The principle for constructing this curve was first presented in 1904 by the Swedish mathematician Helge von Koch. The first step is to draw an equilateral triangle with its side equal to 1. One then starts repeating the procedure by dividing the sides of the triangle into three equal sections and drawing an equilateral triangle, the base of which is the middle section. The procedure ends with the removal of the base i.e. the triangle drawn with a side length of $1/3$ of the section, used in the first step of the algorithm. By drawing an equilateral triangle with side equal to 1, a Sierpiński triangle, described by Waclaw Sierpiński in 1915, can also be formed using the algorithm. The repeatable algorithm procedure begins by dividing sections that form the sides of the triangle in the middle of their length, and drawing four equilateral triangles with a side of $1/2$ comprised in the initial triangle. The next step is to remove the inside of the middle triangle. This procedure is applied to the remaining triangles.

3 Development of Bionic Structural Surfaces Using Mathematical Models

Along with the functional and practical barrier transformation, the shaping of the facade changed its nature. As a result, the external structures cease to be merely a decoration and insulation of the interior of the external factors and conditions, aiming toward an integral formation of the facade as a structural surface. By a structured surface we understand a surface of interconnected elements. The forming of the architectural outer surface "skin" encourages architects to seek bionic solutions. The design based on the patterns found in the world of nature can create sustainable, effective structures, adapted to the surrounding environment, which apply to the same forming processes. An case study analysis was conducted in order to determine the effects described by mathematical models occurring in the natural processes of shaping principles to the design of the bionic facade. The study highlights the use of mathematical models that shape the bionic elevation due to façade design partitions, the spatial form of the object, flat and spatial structural surfaces.

3.1 Facade Divisions

One of the most basic bionic applications of mathematical models is to design the façade divisions, for example, as panels. These divisions serve mainly as an aesthetic function enabling the formation of unusual facades.

An example of such trends is the Minifie Nixon’s *Australian Wildlife Health Centre* project in Melbourne, territory of Healesville Sanctuary, which uses cellular automata. The structure of the facade surface is formed based on cellular automaton, which involves the position of windows and doors, therefore reaching a seemingly chaotic geometry.

Voronoi diagrams were also used in designing the façade divisions of the *Melbourne Recital Center* (Ashton Raggatt McDougall design) to create a visually attractive surface.

The shaping of aperiodic divisions on a facade can be seen on the example of the *Transbay Transit Center* in San Francisco (Clarke Pelli Architects design), where a Penrose Rhombus Tiling was used. The plane tessellation was designed in the form of perforated metal panels which comprise a curved facade.

A similar facade division design was used in the Daniel Libeskind and Cecil Balmond project in London, *The Spiral Extension*. The elevation of the project object involves three types of facade tiles, whose mutual proportions between them were determined in accordance with the golden ratio principle. The use of aperiodic plane divisions in the form of the *Ammann tiling* made it possible to obtain a fractal facade system.
An interesting example based on the research of Jon Conway is also the use of non-periodic Pinwheel tiling, developed by Charles Radin, which was used in the façade division of Federation Square project in Melbourne. This design was used by the Lab Architecture Studio architects.

Another example of the fractal geometry application to divide the façade is the design of the Grand Egyptian Museum façade in Cairo. The Heneghan Peng project assumed the Sierpiński triangle as the prototype algorithm.

3.2 Shaping the Object’s Spatial Form

Mathematical models are being increasingly used in the conceptual stage, especially due to the formation of the spatial form, in which the shape of the object is related to its structure, but is not subordinate to it.

This approach in shaping the architectural body is visible in The Spiral Extension project in London (Daniel Libeskind and Cecil Balmond design), where the golden ratio is the main inspiration. The body of the building was formed on the golden spiral principle composed of polygons. The idea of formation according to a logarithmic spiral was developed by means of fractal geometry. The spiral was derived from several centers, thereby forming various axes along which extends a curve, leading to multiple directions. Folding the flat surface made it possible to obtain a self-supporting spiral, the walls of which stiffen the entire system. As a result, a structural form was obtained that did not require introducing additional support in the interior.

The Velvet State pavilion in Roskilde designed in 2013 by SHJWORKS Architectural as part of Roskilde Festival is an interesting example of the catenary curve in shaping the form of an architectural object. Several shapes form the structure of the pavilion which have been based on the catenary curve as a sequence of curves shown in different sections. As a result, a variety of forms based on the same principle of shaping were achieved. The elevation of the object is made of bent plywood panels with additional elements of polycarbonate, which resulted in obtaining a support structure of the pavilion. The height of the pavilion at the highest point is 7 meters.
3.3 Shaping Flat Structural Surfaces

With every day the contemporary façade design leads to a formation of structured surfaces serving both as a design and aesthetics. Voronoi diagrams can be used for discretizing structured surfaces in order to create natural and interesting objects, designed as classic geometrical forms inspired by nature, which apply natural systems in self-organizing the biological structures.

One such example of using Voronoi tessellation in discretization structured surface can be the Melbourne Recital Center building by architect Ashton Raggatt McDougall, in which the curtain wall façade was modeled after the honeycomb.

The Alibaba Headquarters in Hangzhou, China, designed by Hassell studio is a similar example. Here the Voronoi diagrams are used to create a structure overlapping the atrium and connected to the external structure of the building.

An interesting example of an open pavilion is the London Times Eureka Pavilion designed at the RHS Chelsea Flower Show in 2011. The NEX Architecture project, designed by a landscape architect Marcus Barnett, was based on an algorithm simulating the process of growing trees (L-systems) and Voronoi diagrams. This way the pavilion "emphasizes" the interdependence of the natural ecosystem of science and technology. The inspiration with natural structure divisions and observation of plant growth and development made it possible to create a pavilion of such form.

The 2011 Research Pavilion is an example of Voronoi diagrams in a discretization of a curved structural surface, designed by the Institute for Computational Design and Institute of Building Structures and Structural Design at the University of Stuttgart. The resulting forms were verified in terms of spatial and endurance testing, considering the production capacities. The building was built as a result of a synergistic interdependence in design and construction. As a technical solution, prefabricated wooden panels were proposed, produced using numerically controlled machines (robotic fabrication system), which allowed for the execution of a low weight construction at a relatively high load carrying capacity.
The object structure has the characteristics of biological structures, i.e. diversity, anisotropy and hierarchy.

Figure 5: Structural surface shaped with Voronoi diagrams in Pavilion Eureka Times (left) and the Research Pavilion in 2001 (right). Source: (left): http://www.nex-architecture.com/#/work/projects/times_eureka_pavilion/ Source: (right): <http://www.archdaily.com/200685/icditke-research-pavilion-icd-itke-university-of-stuttgart/>

The SUTD Library Pavilion on the Singapore University of Technology and Design campus, designed by City Form Lab Studio in collaboration with engineers from Arup workshop is an equally interesting design example. Through the use of an arc-shaped catenary curve it was possible to build the roof of the pavilion without the need for intermediate supports. The construction of the pavilion roof covering elements was made of plywood forming triangles, all with the use of steel fasteners.

The use of aperiodic methods of surface discretization is visible in the Melbourne’s flat structural surface facade of Storey Hall building, designed by the architect Asthon R. McDougall. The design employed a Penrose tiling consisting of two kinds of orthorhombic tiles, forming the facade’s glass structure.

3.4 Formation of spatial structures

The biological self-organization structural models found in the natural world also serve as an example in shaping the pattern of a bionic architectural spatial structures. The imitation of patterns found in nature allows for a more efficient design of space-engineering structures as an equivalent of change through evolution, which best aims at adapting a living organism to the environmental conditions. Here, bionic space division methods can be widely used.

For example, the Wheaire-Phelan structure was used in the Water Cube project in Beijing, made in the form of a metal spatial frame that serves as the main support of the object. In addition, the authors of the project, the PTW Architects group, in order to protect the structure against the harmful effects of atmospheric factors and typical for a swimming pool corrosive influence of the atmosphere, designed the ETFE cushions at the same time forming a specific look of the building.

In shaping the optimal spatial structure of an object, Voronoi spatial cells are being increasingly used within conceptual projects. Un Memorial by ACME group may be one such example, which won the third prize in the United Nations Memorial competition. The idea of the project was used to reflect the United Nations structure based on the use of Voronoi tessellation - an organization created with individual nations (i.e. Voronoi cells). The facility provides exhibition spaces, offices and restaurants.
The Vertical Village building concept authored by Yushang Zhang, Rajiv Sewtahal, Riemer Postma and Qianqian Cai architects is a similar example. Through the use of spatial Voronoi cells including the compartmentalization of residential units, it was possible to obtain an optimum construction of the object.

The use of generative design methods based on non-periodic quasicrystal model enabled the generation of an irregular geometry of the Mexico City Manuel Gea Gonzalez Hospital facade. The facade of the building consists of prosolve370e modules designed by Elegant Embellishments (Allison Dring, Daniel Schwaag). An aperiodic net with 5-fold axis of symmetry, consisting of two types of diamonds (analogous to "kites and arrows" Penrose tiling) served as the basis for the Prosolve370e divisional module creation.

In shaping the spatial structures, a non-periodic Pinwheel tessellation tiling with fractal geometry characteristics was used. The geometries seen in the facade of the Federation Square building in Melbourne (Lab Studio concept) were repeated in construction elements.

Figure 6: Left: The facade of the hospital Manuel Gea Gonzalez in Mexico City, where the structure was developed on the basis of aperiodic tiling seen in quasicrystals. Source: http://www.labarchitecture.com/projects/federation%20square.html

Right: The application of Pinwheel tiling forming fractal geometry of structure of Federation Square in Melbourne. Source: <http://prosolve370e.com/pr_torre.htm>

4 Summary

The shaping of bionic structural surfaces begins to pose some importance in contemporary architecture. The inspiration with bionics leads to an interesting type of architecture as an expression of contemporary design trends. The many possibilities to design architecture based on biological structures allow one to create optimized, impressive architectural solutions, efficient also in terms of spatial design. The emerging concepts in the design of modern architecture represent new quality elements formed as a result of the synergy in design solutions.

The increasingly significant impact on the search for shaping the surface of the facade is inspired by biological processes and functional systems. As a result, the elevation should not be a deliverable, but one of the main actors in the design process. The shaping of the modern façade takes on emergent characteristics, enabling the creation of efficient,
sustainable and intelligent facilities. The search in the field of bionic façades is conducted in such a way as to merge various mathematical models at different levels of development of the facade.

The use of models simulating self-organization and natural forming processes is an innovative idea for the development of structural forms, which acts as a structural elevation. It does not, however, apply to all of these aspects. In the case of fractal geometry and cellular automata, they are applied only as a method of a structured surface discretization while shaping the objects on the fractal structure model enables the creation of spatial forms of various bearing systems.

References
GEOMETRIA POWIERZCHNI INSPIROWANA NATURALNYMI SYSTEMAMI WE WSPÓŁCZESNEJ ARCHITEKTURZE

Współczesna architektura coraz częściej jest inspirowana kształtowaniem biomimetycznym. Forma obiektów architektonicznych nie ogranicza się jedynie do estetyki i wzorów spotykanych w Naturze, ale sposób jej kształtowania odwzorowuje naturalne procesy formotwórcze, umożliwiając projektowanie skomplikowanych, optymalizowanych pod różnymi względami struktur przestrzennych. Współczesne tendencje w zakresie kształtowania architektury z uwzględnieniem elementów projektowania bionicznego prowadzą m.in. do poszukiwania optymalnych form, w których zostaje eliminowana zbędnia geometria. Istotną rolę w tym procesie odgrywają narzędzia cyfrowe. Zastosowanie algorytmów i programów komputerowych stanowi o modelowaniu trójwymiarowych struktur przestrzennych i prowadzi do powstania specjalistycznych narzędzi cyfrowych przy pomocy których możliwe staje się generowanie nieregularnych i optymalnych form przestrzennych [1]. W tworzeniu nowej formy obiektów architektonicznych w kształtowania powierzchni strukturalnej coraz częściej stosowane są metody dyskretyzacji powierzchni. Odtwarzanie budowy struktur oraz analiza procesów biologicznych możliwe są dzięki doskonaleniu generatywnych metod projektowania, pozwalających na zgłębienie wiedzy na temat technologii kształtowania elementów ze świata przyrody [2]. Projektowanie oparte na zasadach kształtowania naturalnych struktur wymaga zrozumienia zachodzących procesów i opisania ich za pomocą odpowiednich modeli matematycznych. Zastosowanie modeli matematycznych stwarza możliwość opisywania form ze świata Natury, a w szczególności wzorców formotwórczych. Wśród aspektów formotwórczych zachodzących w przyrodzie opisywanych za pomocą modeli matematycznych na szczególną uwagę zasługują m.in. diagramy Woronoja, ciąg Fibonacciego, czy przekształcenia homologiczne.