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RECIPROCAL SYSTEMS – THE GEOMETRIC TOOL FOR SHAPING TWISTED FORMS OF BUILDINGS

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Abstract: The article discusses the concept of shaping twisted forms of multi-storey buildings on the way of modification the geometrical properties in the systems of mutually supported identical beams used as the structures of individual floor slabs. The analysis was based on the assumption that the slabs are in the shape of regular polygons, the number of beams supporting them is equal to the number of sides of the polygon, each beam is supported on one another beam and exactly one column. The formulas describing geometrical relationships between the parameters characterizing the building envelopes and parameters of structures of the buildings were derived in the paper, on several types of related structural systems. Finally, the paper presents the rules of shaping twisted forms of buildings based on the mentioned analysis, as well as selected examples of them.

Keywords: twisted buildings, reciprocal structures, structural morphology, architectural forms, geometric analysis

1 State of the art

1.1 Twisted buildings

The spectacular form, which was introduced to architecture of high-rise buildings, is the twisted form. All floors of a twisted building are usually identical but each floor is rotated relatively to the previous one by a certain angle. The external walls define helical surfaces [1]. The idea of twisting the building is the result of a search for a distinctive architectural form, as well as a shape resisting the dynamic interaction of wind better than the polyhedral form.

Turning Torso in Malmo, Cayan Tower in Dubai and Evolution Tower in Moscow are the most famous twisted skyscrapers. Stories of them are polygonal and all the floors are rotated relatively to the previous ones by the same angle. Examples of low twisted buildings are Gehry Tower in Hanover and synagogue in Dresden.

1.2 Reciprocal systems

Structural elements may be linked together in this way that each one of them supports the next element and is supported by the previous one (Fig. 1a). A structural system, which is distinguished by such a feature is called reciprocal system. The reciprocal systems have been known since the Middle Ages from drawings of Villard de Honnecourt, Leonardo da Vinci and Sebastiano Serlio [2]. According to [3], the important requirement for the reciprocal system is that every external support should be located along the span of element, not in the junction point (Fig. 1b).

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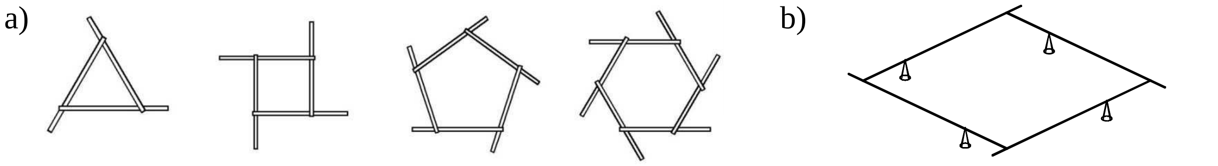


Figure 1: Reciprocal systems: a) the essence, b) the arrangement of external supports

2 Reciprocal systems of beams as structures of floor slabs

2.1 Assumptions

The optimal way to transfer gravity loads from the floors of building to the ground, is to carry them via vertical structural elements. In twisted buildings, such assumption requires that supports were arranged on all floors uniformly in the plan, but non-uniformly with respect to the individual floor slabs, rotated to each other. The presented solution consists in using vertical columns supporting the sets of linked beams and acceptance the principle that in different floors the beams are supported with columns at points spaced differently along the beam lengths. The sets of beams arranged in this way are typical reciprocal systems (Fig. 2).

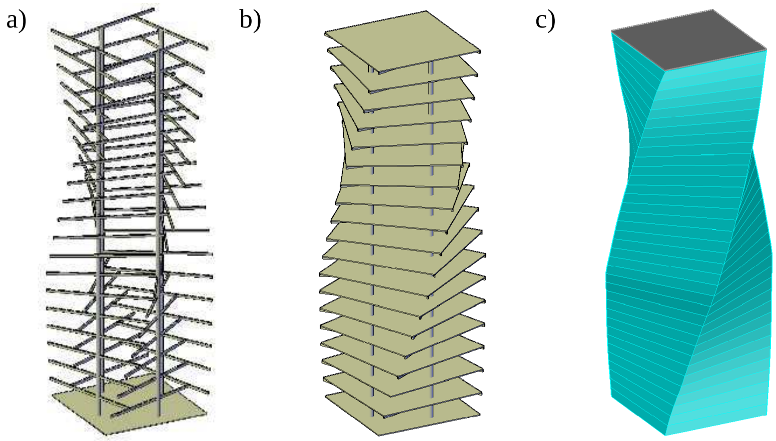


Figure 2: The concept of shaping twisted buildings using reciprocal systems of beams: a) structural skeleton, b) floor slabs, c) external view

The solution is proposed in few variants. In all of them slabs are in a shape of a regular polygon, the number of columns and the number of beams supporting the slab are equal to the number of sides of the polygon, which means that each beam is based exactly on one column and one other beam, and it constitutes the support for the another beam by itself.

A system of beams connected at the ends, supporting the floor slab along its circumference is the variant from which the consideration has been begun (Fig. 3a). In the figure, the beams supporting two next slabs are indicated with a continuous thick line, while a thin dotted line marks the polygon with vertices defined by the arrangement of columns. The beams belonging to the next stories of the building are rotated about axes of columns by different angles. The consequence is diversity of lengths of beams and thus sizes of slabs on each floor.

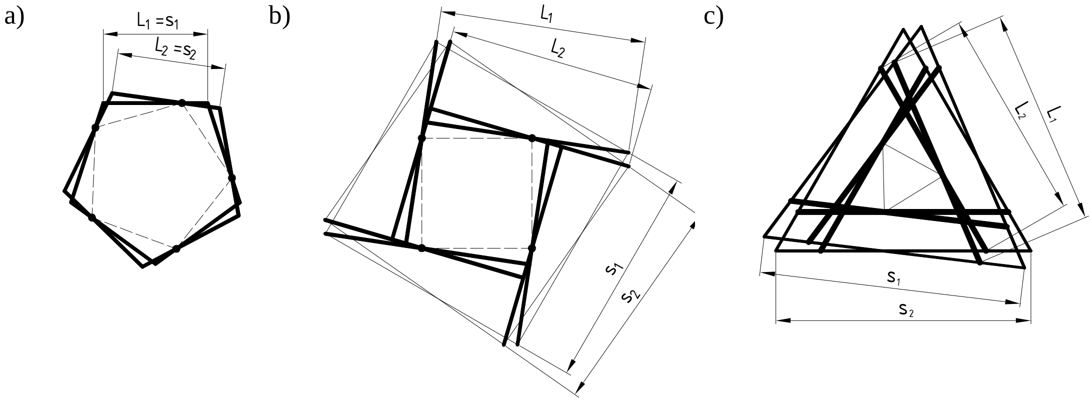


Figure 3. Reciprocal systems of beams: a) without cantilevers, b) one-sided cantilevered, c) double-sided cantilevered

The next type of beam systems (Fig. 3b) is the result of side extension of the beams outside the points of connection. Because the lengths of cantilevers are not limited, the sizes of the following floor slabs may be formed freely, including they can be equal. Another modification of the beam system consists in extension the beams on both sides of the connection points. The lengths of the left and right cantilevers may be the same or different. In this article, only the systems limited to equal lengths of both cantilevers are analyzed (Fig. 3c). In all the mentioned variants, systems of beams may be formed on the basis of any regular polygon.

2.2 Geometric analysis

The trigonometric description of the correlation between the architectural form and the structural system of the building is the main aspect of the considered issue. It has been assumed that parameters describing a system of floor beams, related to the architectural form, are the side length of the floor slab (s), as well as the angle between the side of the slab and the side of the polygon defined by columns (φ_{cs}). The parameters describing the geometry of the structural system are: the distance between next columns (c), the beam length (L) and the angle between beams and sides of a polygon defined by columns (φ_{cb}). Auxiliary parameters are the lengths l_1, l_2 and l_3 specifying the division of the total beam length L by the point of support and the point of connection with adjacent beams. All the formulas are derived for any polygon in the functions of angle ν_n, (1), in which n is the number of angles in the polygon.

$$v_n = 180 - \frac{360^\circ}{n} \tag{1}$$

Shaping the twisted building can be subordinated to optimizing architectural form or structure. With regard to the external walls, the optimal form of the building is characterized by constant dimensions of the floor slabs, and a constant angle between edges of the slabs of two next floors. Structural optimization should be aimed at the unification of beam-column connections by providing a constant change in the angle of rotation of the beams around the column. Unification of beam lengths is less important in real structure, although it may inspire to carry out the compositional experiments in model scale. The parameter that must remain constant for all floors is the distance c between the columns.

If the variable for structures formed according to the Figure 3a were $\varphi_{cb} = \varphi_{cs}$, the lengths of segments l_1 and l_2 and total beam length L (Fig. 4a), are expressed by the formulas

(2), (3) and (4). For the back way of forming, consisting in the initial determination of the dimension L = s, the formula for $\varphi_{cb} = \varphi_{cs}$ is expressed by equation (5).

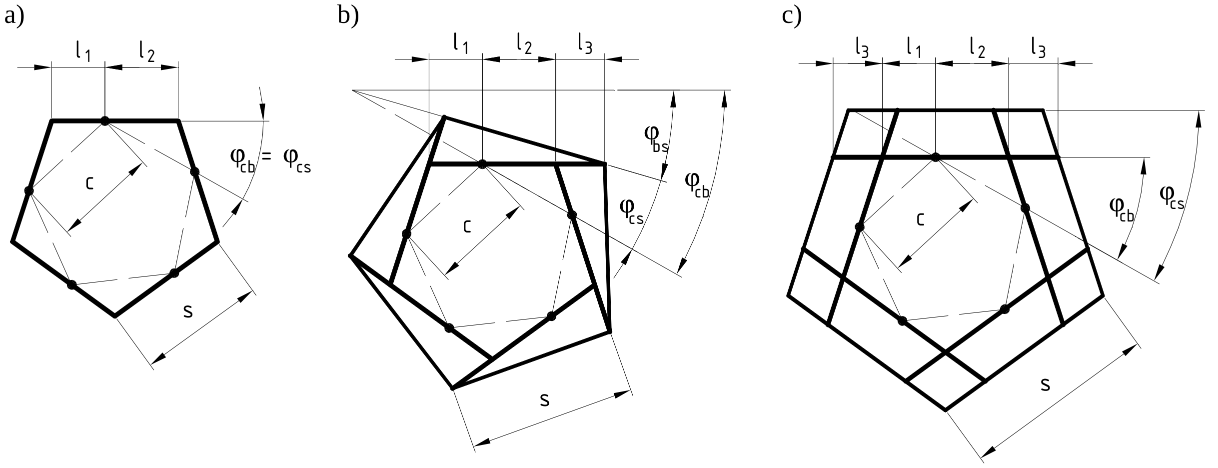


Figure 4: The parameters defining reciprocal systems of beams according to: a) Fig. 3a, b) Fig 3b, c) Fig. 3c

$$l_1 = \frac{c \sin \varphi_{cb}}{\sin v_n} \tag{2}$$

$$l_2 = \frac{c \sin(\varphi_{cb} + v_n)}{\sin v_n} \tag{3}$$

$$L = l_1 + l_2 = \frac{c \sin\left(\varphi_{cb} + \frac{v_n}{2}\right)}{\sin \frac{v_n}{2}} \tag{4}$$

$$\varphi_{cb} = \varphi_{cs} = \sin^{-1}\left(\frac{s}{c} \sin \frac{v_n}{2}\right) - \frac{v_n}{2} \tag{5}$$

In the case of structures shaped according to Fig. 3b, values of the parameters L, s, φ_{cb} and φ_{cs} are different. Taking any two of them as variables, the structures could be formed in six different ways. Assuming that the aim is unifing the outer casing, ie. acceptance of parameters s and φ_{cs} as variables, the left dimensions are determined by formulas (6)÷(11). Assuming L and φ_{cb} as variables, other dimensions are described using formulas (12)÷(15).

$$l_1 = \frac{c}{\sin v_n} \frac{s \sin\left(\varphi_{cs} + \frac{v_n}{2}\right) - c \sin \frac{v_n}{2}}{\sqrt{c^2 + s^2 + 2cs \cos(\varphi_{cs} + v_n)}} \tag{6}$$

$$l_2 = \frac{c}{\sin v_n} \frac{s \sin\left(\varphi_{cs} + \frac{3}{2} v_n\right) + c \sin \frac{v_n}{2}}{\sqrt{c^2 + s^2 + 2cs \cos(\varphi_{cs} + v_n)}} \tag{7}$$

$$l_3 = \frac{s}{\sin v_n} \frac{s \sin \frac{v_n}{2} - c \sin\left(\varphi_{cs} + \frac{v_n}{2}\right)}{\sqrt{c^2 + s^2 + 2cs \cos(\varphi_{cs} + v_n)}} \tag{8}$$

$$L = l_1 + l_2 + l_3 = \frac{s}{\sin v_n} \frac{c\sin\left(\varphi_{cs} + \frac{3}{2}v_n\right) + s\sin\frac{v_n}{2}}{\sqrt{c^2 + s^2 + 2cs\cos(\varphi_{cs} + v_n)}} \tag{9}$$

$$\varphi_{cb} = \tan^{-1} \frac{s\sin\left(\varphi_{cs} + \frac{v_n}{2}\right) - c\sin\frac{v_n}{2}}{c\cos\frac{v_n}{2} + s\cos\left(\varphi_{cs} + \frac{v_n}{2}\right)} \tag{10}$$

$$\varphi_{bs} = \tan^{-1} \frac{s\sin\frac{v_n}{2} - c\sin\left(\varphi_{cs} + \frac{v_n}{2}\right)}{c\cos\left(\varphi_{cs} + \frac{v_n}{2}\right) + s\cos\frac{v_n}{2}} \tag{11}$$

$$s = \sqrt{\left[2L\cos^2\frac{v_n}{2} - \frac{c\sin\left(\varphi_{cb} + \frac{v_n}{2}\right)}{\sin\frac{v_n}{2}}\right]^2 + L^2\sin^2 v_n} \tag{12}$$

$$l_3 = L - \frac{c\sin\left(\varphi_{cb} + \frac{v_n}{2}\right)}{\sin\frac{v_n}{2}} \tag{13}$$

$$\varphi_{cs} = \varphi_{cb} - \sin^{-1}\left[1 - \frac{c\sin\left(\varphi_{cb} + \frac{v_n}{2}\right)}{L\sin\frac{v_n}{2}}\right] \tag{14}$$

$$\varphi_{bs} = \sin^{-1}\left[1 - \frac{c\sin\left(\varphi_{cb} + \frac{v_n}{2}\right)}{L\sin\frac{v_n}{2}}\right] \tag{15}$$

In the structures shaped as shown in Fig. 3c there is a relationship $\varphi_{cb} = \varphi_{cs}$. The lengths of sections l_1, l_2 and l_3 and the total length of the beam L are expressed by the formulas (2) and (3) and (16) and (17).

$$l_3 = \frac{s\sin\frac{v_n}{2} - c\sin\left(\varphi_{cs} + \frac{v_n}{2}\right)}{4\sin\frac{v_n}{2}\cos^2\frac{v_n}{2}} \tag{16}$$

$$L = \frac{s\sin\frac{v_n}{2} - c\cos v_n \sin\left(\varphi_{cs} + \frac{v_n}{2}\right)}{2\sin\frac{v_n}{2}\cos^2\frac{v_n}{2}} \tag{17}$$

3 Spatial shaping of twisted structures by differing systems of beams in next floors

The relationships that have been derived are a tool for shaping the structures of high-rise buildings that spatial arrangement can be far different. Shaping the various architectural forms is possible, as well as the formation of alternative structural systems related to the same form.

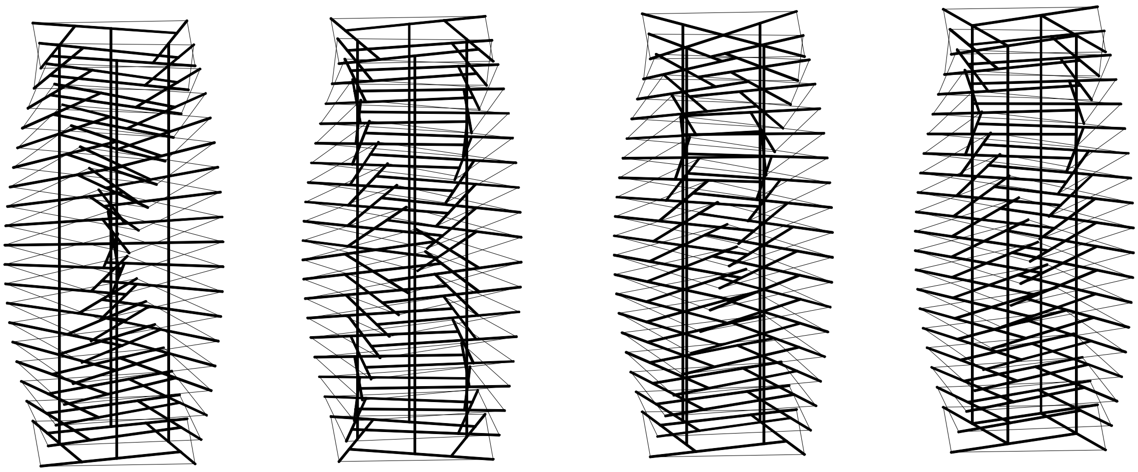
The ability to differentiate spatial structure results from the lack of strict restrictions in the selection of beam systems for each floor of the building. Moreover, all beam systems may assume configurations shown in the previous drawings or under their mirror images.

Differences resulting from the use of mentioned beam systems are as follows:

- modification of the structure or architectural form of a building, constructed with systems of beams that are devoid of cantilevers (Fig. 3a and 4a) is not possible, except for changing the position of the smallest (largest) storey along the height of the building,
- buildings based on systems of one-sided cantilevered beams (Fig. 3b and 4b) may be modelled both with storeys equal or unequal with the use of structural systems, characterized by different spatial configurations; in this case, systems of beams specified for the $\varphi_{cs} \in [0, 90°]$, or for the values selected from custom interval, which of the range is 45°, and systems symmetrical to them, may be assigned to the individual floors of a building,
- using the systems of double-sided cantilevered beams (Fig. 3c and 4c) results in a lack of possibility for modifying the spatial configuration, except that the particular floors may be smaller or greater thanks to a choice of cantilever lengths.

The range of architectural forms possible to shape on the base of an application of reciprocal systems of beams as the floor structures is limited only by the imagination of an architect. By combining different systems in one building, as well as individual differentiation of geometrical parameters of particular floors, or the direction of rotation, the complex and unconventional forms can be modelled. Flexibility in the method is presented in the Fig. 5 and Fig. 6. The first figure presents variety of systems correlated with the same external form of a building, the second one - a little artistic vision of “a twisted town”.

a)



b)

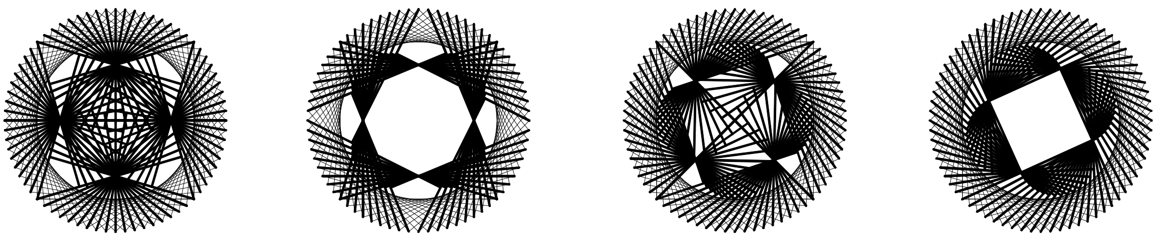


Figure 5: The use of different structural systems in twisted buildings of the same external form, which are modelled with the use of systems of one-sided cantilevered beams: a) axonometric views, b) top views

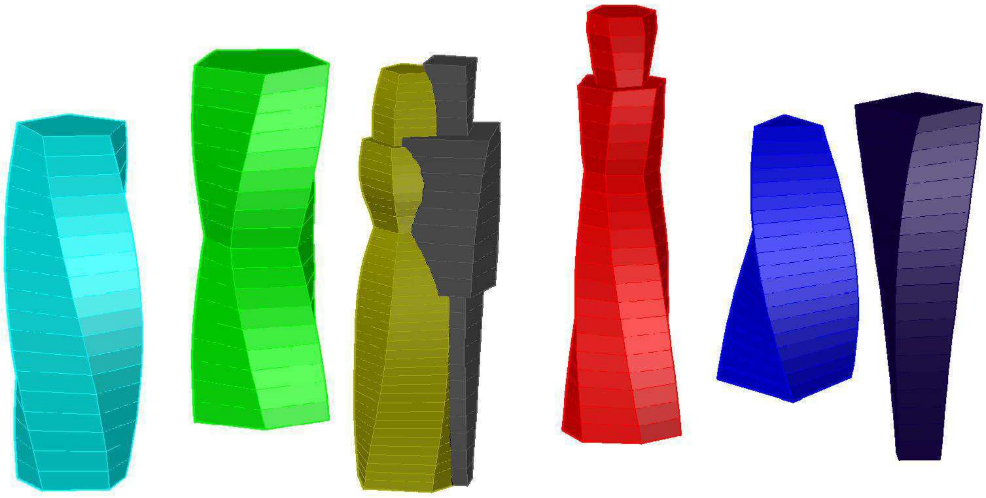


Figure 6: Exemplary forms of twisted buildings obtainable with the use of reciprocal systems of beams as floor structures

4 Conclusion

As a result of the considerations described in this article, such structural system for twisted buildings has been invented that allows to transfer gravity loads only via the vertical columns and thereby makes the areas of each floor convenient for freely planning their functional arrangement. Practical implementation of solutions that relevance has been proven on examples of geometric models, requires consideration of the static aspect, including taking into account the wind load, significant in multi-storey buildings. The author will try to rationalize the system in such a way that will not deprive it of existing advantages. It is also foreseen to extend the analysis on multi-reciprocal systems of beams arranged on grids based on squares as well as on equilateral triangles and hexagons.

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ZESPOŁY ELEMENTÓW WZAJEMNIE PODPARTYCH - GEOMETRYCZNE NARZĘDZIE KSZTAŁTOWANIA BUDYNKÓW O FORMACH SKRĘCONYCH

W artykule omówiono koncepcję kształtowania form skręconych budynków wielokondygnacyjnych w oparciu o modyfikację właściwości geometrycznych w systemach wzajemnie podpartych identycznych belek zastosowanych jako konstrukcje stropów. Analizę oparto na założeniu, że płyty stropowe mają kształt wielokąta foremnego, liczba podpierających je belek jest równa liczbie boków wielokąta, zaś każda z belek jest podparta na jednej innej belce oraz na jednym słupie. Wyprowadzono związki geometryczne określające miary kątów pomiędzy bokami wielokąta wyznaczonego przez słupy a bokami płyty stropowej, w funkcji zmiennych wyrażających wzajemne położenie belek oraz rozmieszczenie słupów. Na podstawie analizy tych zależności przedstawiono na przykładach technicznie racjonalne oraz wizualnie interesujące formy skręcone budynków wysokich.

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