APERIODIC TESSELLATIONS IN SHAPING THE STRUCTURAL SURFACES IN THE CONTEMPORARY ARCHITECTURE

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Abstract. The contemporary trends in the division of structural surfaces used in architectural forms refer to the known principles of mathematics and geometry - a simplified record of the natural world that surrounds us. A tessellation of a repetitive geometric design plays an important role in the discretization of the curvilinear architectural forms in filling out areas or spaces. In the process of determining the distribution of the division of geometrically irregular and complex surfaces, aperiodic tessellation are becoming increasingly used instead of polygonal tessellation, whose characteristic fractal structure is closer to the technology of Nature and can lead to more efficient engineering solutions. In the development of contemporary architecture, the inspiration with bionics is an interesting trend expressing, among others, in imitation of biological processes for the development and construction of organic structures. The purpose of these actions is to search for forms of originality, whose logic of shaping focuses particularly on effectiveness in the use of material and energy. The development of digital tools, especially through algorithms for 3D modeling programs but also through morphogenesis, has enabled generating complex systems.

The use of aperiodic tessellation in the era of generative design methods provides new, creative tools in shaping flat and spatial rod structures. One such example in the generative modeling is the use of geometry in non-periodic divisions is Danzer Script, which allows for the design of systems based on a seemingly chaotic structure of the quasi-crystal. This method is interesting because of a characteristic non-periodical element of the structure resulting from its symmetry and manifested in embodiments, structure and physical properties of quasi-crystals. In a time of a universal algorithmization of an architect's working tools, the use of unobvious, chaotic bionic construction structures creates new opportunities for interdisciplinary and creative architectural designs. The digital tools for modeling rod structures and generating structural divisions based on aperiodic tessellation is a synergistic action to seek new architectural and structural solutions.

Keywords: aperiodic tessellations, fractal geometry, quasi-crystal

1 Introduction

In contemporary architecture there is an interesting trend of searching for optimal shapes inspired by the natural world which eliminate the process of unnecessary geometry. Along with the development bionic trends in architecture is a growing need to improve the digital shape optimization tools grows. Based on algorithms and spatial 3D modeling programs, specialized tools generating digital and simultaneously optimal, irregular spatial forms can be created [2]. Improving the generative design methods is one of the elements that allows for

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the exploration of Nature's technology by reconstructing the bionic structures and processes of the biological development [3].

The aim of the optimization, which appears in the process of shaping the structural forms and their implementation is to determine the size of the internal static quantities subjected to static and dynamic loads. In this case, the most commonly used are numerical methods, which discretize (divide) the structure into so-called finite elements [4].

Dividing structural architectural forms with regular figures can be done by tesselation. Periodic divisions for complex surfaces or spaces can obtained as a result of geometric surface transformations such as translations, rotations and reflections, and by folding [6]. Periodic tessellation described so far are mostly composed of regular polygons such as equilateral triangle, square, hexagon, octagon and decagon [1]. The characteristic surface of the roof of Sydney Opera House is an example of a polygonal tessellation used in architecture. Jorn Utzon's spectacular design (implemented in years 1956-1973) was the first project, which used a spherical segment approximation of surfaces with different radii [1].

Aperiodic (chaotic) plane tessellation is a separate way of determining surface nodal divisions, the first examples of which were created in the 1970s by the British mathematician, Roger Penrose. Aperiodic patterns are characteristic for their inability to be converted by a translation, and have a 5-fold rotational symmetry, which also occurs in the construction of the quasi-crystals' structure [5]. Initially, aperiodic divisions were seen as an exception to the classic, regular tessellation, however, as the general knowledge expanded, the aperiodic divisions were recognized as an important element of the natural way of development and change in the universe. Now, as the age of digitization and the development of bionic trends unfolds, the fascination with aperiodic tessellations increases, as an interesting tool in shaping optimal technical architectural structures.

2 The application of the aperiodic tessellation modeling of spatial forms on selected examples of the contemporary architecture

2.1 Aperiodic tessellation

An example of aperiodic tessellation is a Penrose tiling (described by Roger Penrose), consisting of four elements that may result in different designs depending on the arrangement (Fig. 1a). There are uncountable ways to divide the area with the aperiodic Penrose tessellation, however, one of the most common types of tessellation is commonly known as 'kite' and 'dart' (Fig.1b). The tessellation is composed of two rhomboids (formed with two gold triangles) the side of which is of equal length and is inclined at different angles (acute angle rhomboids of 36° and 72°). The sole rule for building a tessellation is that the two osculating tiles do not form a parallelogram. This tessellation scheme was used in Melbourne's Storey Hall (Ashton Raggatt McDougall project) to generate the divisions of the facade and selected interior elements.

Two architects from the Lab Architecture Studio (Peter Davidson and Don Bates) applied the Penrose non-periodic system in Melbourne's Federation Square (1997-2001) [1]. The subject of the modelling analysis was a building complex with an adjoining square, forming a unique urban and architectural structure, designed according to the idea of fractal geometry. The aperiodic tessellation is applied to sections of building facades and elements in the grid forming a multicolored surface of the square. The aperiodic pinwheel tiling tessellation is also the main theme for the façade sectioning developed by Charles Radin based on the findings of John Conway [7]. The tiling of this tessellation arises from dividing a right triangle with side lengths of 1, 2 and $\sqrt{5}$ into five isometric copies, by scaling and rotating the individual elements. The pinwheel tiling structure was based on a recursive right triangle delamination, the resulting pattern of which forms an image of the fractal. These characteristics of the pinwheel tiling make the design very popular among architects, including the three-dimensional rod system modelling, called quaquaversal tiling.

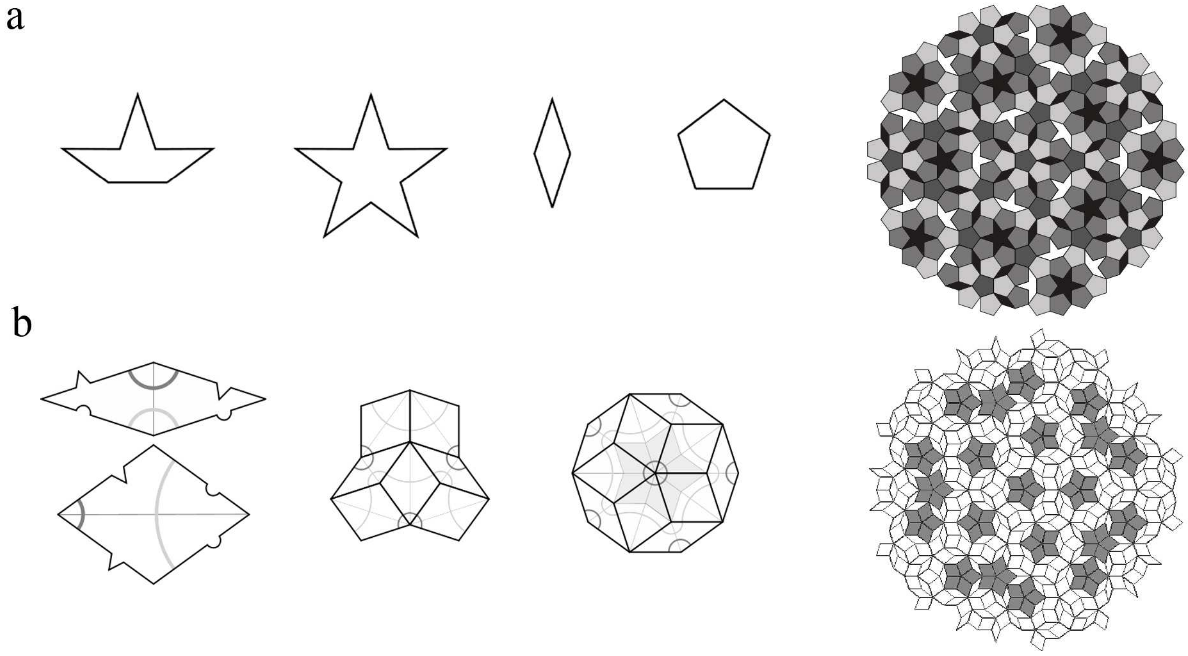

Figure 1: Penrose tessellation; a) four repetitive elements and a tessellation design sample; b) two repetitive elements ('kites' and 'darts') and a tessellation sample (gray color indicates the repetitive theme that acts analogous to the diffraction of quasi-crystals and 'grows' as the distance from the center increases. *Source*: a) copyright materials b) "Wikipedia – The Free Encyclopedia", *Penrose tiling*, [accessed on May 13, 2014] < http://en.wikipedia.org/wiki/Penrose_tiling >

Other aperiodic divisions are applied in modern architecture as well, the so-called Ammann tiling, which can be combined with the geometry of the golden spiral [5]. The Spiral - Extension to the Victoria and Albert Museum , designed by Daniel Libeskind is an interesting example of architecture that uses aperiodic divisions on the façade that are consistent with the geometry shaping the structural form. The project set to expand the existing museum was made in collaboration with structural engineer, Cecil Balmond. The proposed spiral form by the architect is a contemporary response to the rapid transformation of museology. The distinctive shape of the object helps 'build the tension' while visiting the exhibition, at the same time maintaining the individuality of space and allowing them to work independently. The ideological form of the spiral reflects the mathematic and geometric principle of the golden spiral, which was used both to build a conceptual body of the building and the facade sectioning. One of the characteristic of the golden spiral is a geometry expressed through a fractal image, which is a specific example of a logarithmic spiral. The fractal geometry of the infinite spiral structure was therefore an expression of the ideological assumptions of designers of a dynamic form of the building that evolves over time. The spiral was derived from a number of centers and runs along different axes, setting many directions, roads and exhibition routes and spaces of different kinds. Based on the spiral's geometry, the structural form of the object was shaped on an irregular flat surface, which has characteristic spiral folds around the perimeter of the quadrilateral. The angles of the folds and the lengths have been specially designed to elevate the self-supporting upward spiral, whose subsequent folds stiffen the lower surfaces of the folded wall systems by means of braces. The obtained structure forms a solid spatial arrangement, free of any additional support in the interior. Due to the dynamic load of the proposed form, the structural model was subjected to static analysis and simulations, which resumed the search for the most optimal spiral geometry and its progress. Thanks to the division of Ammann's tiling, the spiral continued on the facade. The structure of aperiodic divisions based on the spiral made it possible to provide a fractal image, the nature of which varies smoothly as a result of the iteration compaction. In addition, depending on the function, intensifying the divisions based on a spiral of self-similar figures regulates the translucency of the compartment. For political reasons the project by Daniel Libeskind and Cecil Balmond was not realized, however, is an interesting example of the search for unusual tectonic solutions in architecture.

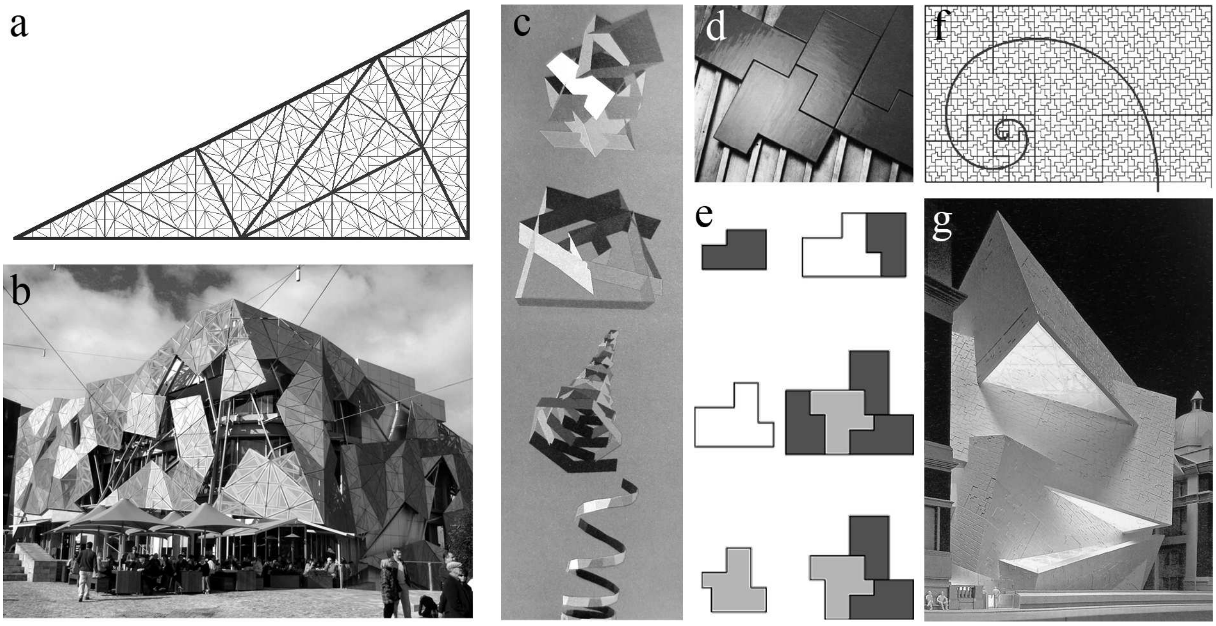

Figure 2: *Federation Square*, design by Architecure Lab Studio, Melbourne; a) aperiodic *Pinwheel tiling* divisions, b) view of the building with its characteristic façade sectioning; The Spiral – Extension to the Victoria and Albert Museum, design by Daniel Libeskind and Cecil Balmond, London; c) a method of forming a solid object using the golden spiral geometry; d) Ammann tiling model used for facade sectioning; e) three repetitive tessellation elements and a sample composition of these elements; f)- the geometry of aperiodic divisions in relation to the golden spiral; g) a model that shows the body of the building. *Source*: a) Xah Lee, "Algorithmic Mathematical Art", *L-systems, Turtle Graphics*, 2004, [accessed on May 13, 2014], < http://xahlee.info/math/algorithmic_math_art.html > b) "GAA: Feature", *Federation Square & downtown*, [accessed on May 13, 2014] < http://www-personal.umich.edu/~drossz/oz/melbourne.htm > c, d, f, g) Balmond Cecil, Smith Jannuzzi, *"Informal"*, str 189-216, Wyd: PRESTEL, 2002; e) "Tiling Encyclopedia", *Ammann A3*, [accessed on May 13, 2014] < http://tilings.math.unibielefeld.de/substitution_rules/ammann_a3 >

2.2 Aperiodic structure model of quasi-crystal

Three-dimensional structures called quasi-crystals, discovered by Dan Shechtman in 1984 in a cooled aluminium-manganese alloy also show the characteristic properties of two-dimensional aperiodic tessellation [7]. The seemingly regular structure of quasi-crystals does not arise as a result of filling the space with identical primitive cells, but as a result of aperiodic tessellation, which provides a 5-fold and higher symmetry axes, unusual for the construction of the classical crystal structure [5]. (Figure 3c) Quasi-crystal atoms arrange in a quasi-periodic manner, or 'almost repetitive' one, whereas the (simplified) spatial structure of the building is formed with different geometric solids, such as rhomboedrons, dodecahedrons, icosahedrons, rhombic triacontahedrons [7].

Based on the aperiodic quasi-crystals' body structure, specialized computer programs are made for modeling the spatial architectural forms, such as the Danzer Script. The program was based on the theoretical studies of Louis Danzer and generates a spatial, aperiodic plate composed of four tetrahedral solids (Tetrahedron A, B, C, K), wherein the edge lengths equal a multiple of the golden number (Fig. 3a). The edges that form a tetrahedron are divided into three classes; the geometric transformations and the rules for matching and collating the individual elements are all based on those classes. In recent years, many artists and engineers were inspired by the Danzer tiling structure, for example the Solid Void by Cecil Balmond at the "Graham Foundation For Advanced Studies in Fine Art" exhibition (Chicago 2008-2009) (Figure 3b).

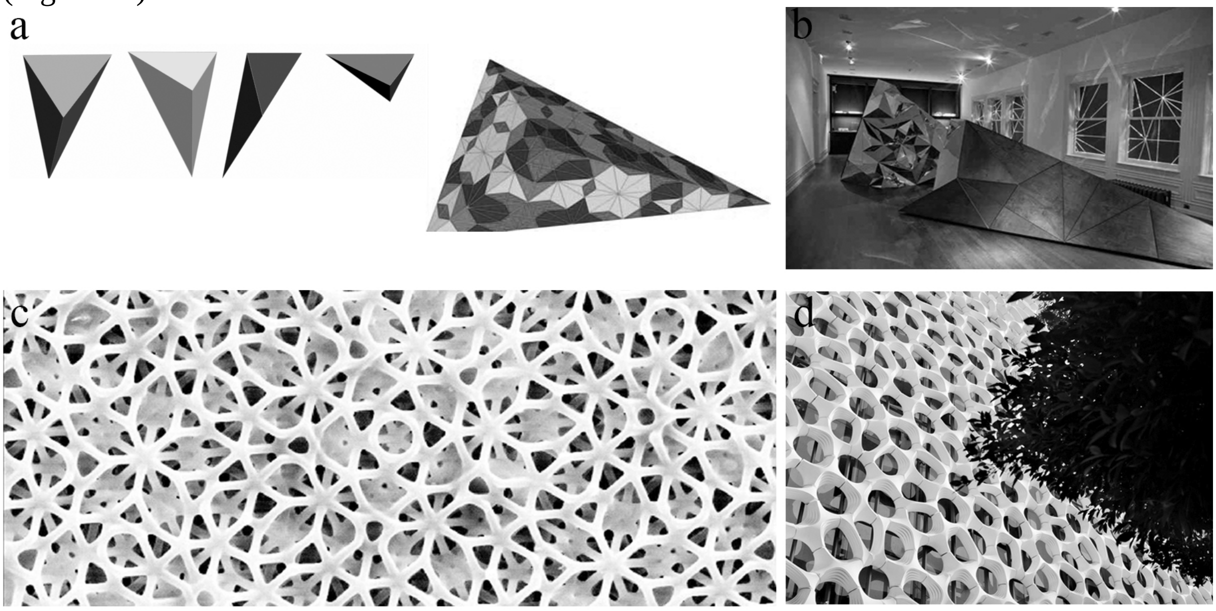

Figure 3: Dänzer Script; a) a set of A, B , C, and K (left) tetrahedron forming an aperiodic, three-dimensional plate (right) according to the Dänzer script; b) 'Solid Void' exhibition by Cecil Balmond, inspired by Dänzer tiling; Quasicrystals; c) quasi-crystal structure; d) façade of the Manuel Gea Gonzalez hospital in Mexico City with visible divisions inspired with quasi-crystals *Source*: a**)** Charalampous Charis, "Responsive spatial growth of the Danzer packing", [Access on May13, 2014], Access on World Wide Web: < http://eprints.ucl.ac.uk/4982/1/4982.pdf> b) Becker Lynn, "Cecil Balmond and the Bonfire of the Vanities", [Access on May 13, 2014], Access on World Wide Web: <http://www.lynnbecker.czom/repeat/burnham/burnhampavilions/burnham_pavilions_chicago_hadid_unstudio.htm > c**)** Optics of Complex System, Quasicrystals, [Access on May 13, 2014], Access on World Wide Web: < http://www.complexphotonics.org/quasicrystal.html> d) Elegant Embellishments, Prosolves370e. [Access on May 13, 2014]. Access on World Wide Web: <http://www.prosolve370e.com>

The use of generative design methods based on non-periodic quasi-crystal model made it possible to generate the irregular facade geometry of the Manuel Gea Gonzalez Hospital in Mexico City. The façade of the building consists of prosolve370e modules designed by Elegant Embellishments (Allison Dring, Daniel Schwaag). The prosolve370e is a three-dimensional, decorative architectural module used for reducing the amount of pollutants in the air by means of photocatalytic pollution-fighting technology (light-activated titanium dioxide coated modules). An aperiodic grid with a 5-fold axis of symmetry, consisting of two types of diamonds (analogous to Penrose's Tessellation 'kites' and 'darts') was the basis for the creation of Prosolve370e division modules. Organic divisions forming an irregular pattern consisting of two types of tiles could be generated on the grid's basis.

Prosolve370e modules can also be used as an element of interior design, as in the case of the Interior Decorative Ceiling and Walls project of the Enex100 building, designed by the COX Australia studio architects from Perth, Australia (2009). The modules form a topographic structure of the ceiling and wall elements consisting of approximately 750 items with a total area of 800 m2. The above mentioned modules form a double-sided screen in the Al Bustan hotel lobby in Abu Dhabi.

3 Summary

The contemporary search in the field of surface and space discretization is increasingly used in architectural design. The aperiodic tessellations are particularly interesting and moreover allow for the development of harmonious, non-periodic divisions. The scientific discoveries in the field of three-dimensional aperiodic structures offer new opportunities for developing biomimetic structures and fractal systems. The application of new methods for the allocation of space can be an important element in shaping both the façade and the load-bearing structures of buildings.

Through the use of generative design methods, the use of architectural geometric objects in shaping the described form was possible. The development of digital tools creates better conditions for analyzing and building more complex structures and systems of elevation, using, among others, the aperiodic divisions.

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TESSELACJE APERIODYCZNE W KSZTAŁTOWANIU POWIERZCHNI STRUKTURALNYCH WE WSPÓŁCZESNEJ ARCHITEKTURZE

Współczesne tendencje podziału powierzchni strukturalnych stosowanych w formach architektonicznych odnoszą się do poznanych zasad matematyczno-geometrycznych, będących uproszczonym zapisem otaczającego Nas świata przyrody. W dyskretyzacji krzywoliniowych form architektonicznych istotną rolę odgrywa tesselacja polegająca na wypełnianiu powierzchni lub przestrzeni powtarzającym się motywem geometrycznym. W procesie wyznaczania podziału dla nieregularnej i złożonej geometrycznie powierzchni coraz częściej zamiast tesselacji poligonalnych wykorzystuje się tesselacje aperiodyczne, których charakterystyczna budowa fraktalna jest bliższa technologii Natury i może prowadzić do bardziej efektywnych rozwiązań inżynierskich. Ciekawy kierunek w rozwoju współczesnej architektury to inspiracje bioniką, wyrażające się m.in. w naśladowaniu biologicznych procesów rozwoju i budowy struktur organicznych. Celem takich działań jest poszukiwanie oryginalnych form, których logika kształtowania ukierunkowana jest szczególnie na efektywność w zużyciu materiału i energii. Rozwój narzędzi cyfrowych, w tym szczególnie algorytmizacja programów do modelowania 3D umożliwiły generowanie skomplikowanych układów, w tym także na drodze morfogenezy.

W dobie generatywnych metod projektowania, wykorzystanie tesselacji aperiodycznych dostarcza nowych, twórczych narzędzi w kształtowaniu płaskich i przestrzennych struktur prętowych. Takim przykładem zastosowania geometrii nieokresowych podziałów aperiodycznych w generatywnym modelowaniu jest Skrypt Danzer'a, pozwalający na projektowanie układów w oparciu o pozornie chaotyczną strukturę quasi-kryształu. Interesująca w tej metodzie jest charakterystyczna budowa elementu nieperiodycznego wynikająca z jego symetrii, a przejawiająca się w postaciach, strukturze oraz właściwościach fizycznych quasi-kryształów.

W dobie powszechnej algorytmizacji narzędzi pracy architekta, wykorzystanie nieoczywistej, chaotycznej budowy struktur bionicznych stwarza nowe możliwości w interdyscyplinarnymi twórczym projektowaniu architektury. Cyfrowe narzędzia do modelowania struktur prętowych oraz generowanie podziałów powierzchni strukturalnych w oparciu o tesselacje aperiodyczne to także działania do poszukiwań synergicznych rozwiązań architektoniczno-konstrukcyjnych.

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