MONGE METHOD IN CREATING REGULAR POLYHEDRONS MODELS

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Abstract: Physical face (paper) models of polyhedrons, particularly Platonic solid models, are formed by drawing the nets before them. Other construction requirements makes us the task to run a solid (physical or virtual) model. Beside the solid faces their mutual angles or angles to the selected plane is needed to know. Sometimes other parameters such as a distance parallel faces is good to know. If we choose the geometric way, it does not perform calculations that can be performed using a AutoCAD program. Then turns out to be a very helpful Monge method. The paper contains a proposal to carry out such a task.

Keywords: polyhedron, face model, platonic solid, Monge method, primitive

1   Introduction

The Monge method in many cases may be a good starting point for modelling three-dimensional objects using standard CAD software [4]. Using this method, we find all the parameters (angles and segments) needed to perform the spatial structure. This is very important because spatial modelling can frequently be associated with the operation of the space consisting in the skillful handling of the available primitives; with the action which is already widespread in the design: from 3D do 2D, where we first make a 3D model and then find the appropriate plane views. This action is correct, as long as we have all the necessary primitives and possible operations on them. 3D modelling is the virtual equivalent of the classical implementation of a geometric paper model, which so far has been the ultimate verifier of solution's validity. The desirability of the theoretical basis of the Monge method is often discussed [5,6] and natural examples of use are its verification [2,3].

2   Create virtual models of the platonic solids

The classic net method of creating (bonding) regular polyhedrons requires only knowledge of the dimension of the side face of the solid (see Fig.1ab, **h**ence there is no need for other geometric parameters of solids, such as an angle between adjacent faces, distance between parallel faces (if any exist).
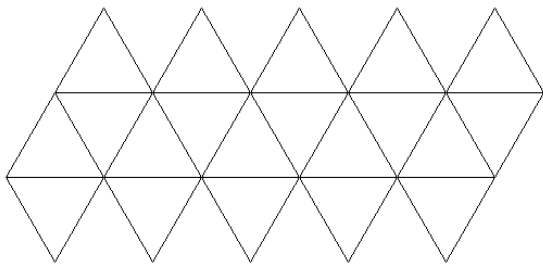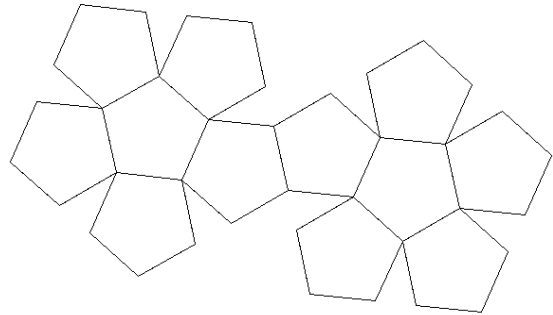
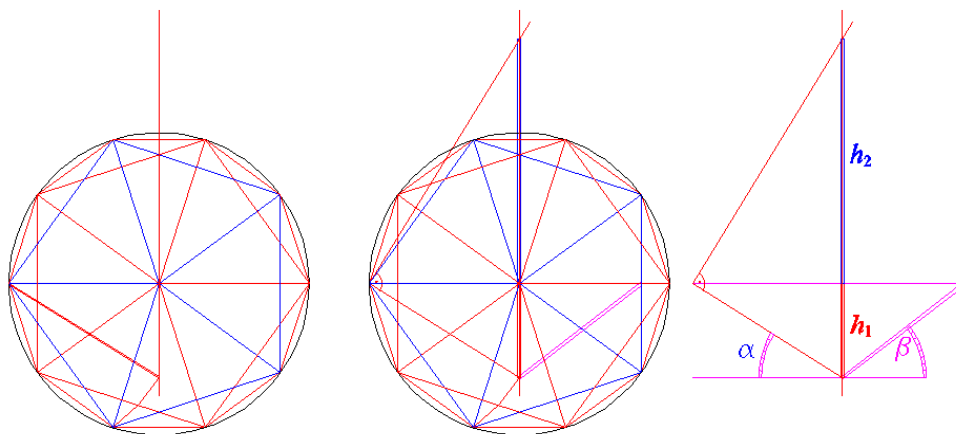Figure 1a: Icosahedron net

Figure 1b: Dodecahedron net

Figure 2: The horizontal projection of a icosahedron and the manner of determination of two angles: α - the angle between an edge and the horizontal projective plane, β - the angle between a face and the horizontal projective plane and the diagonal of the icosahedron
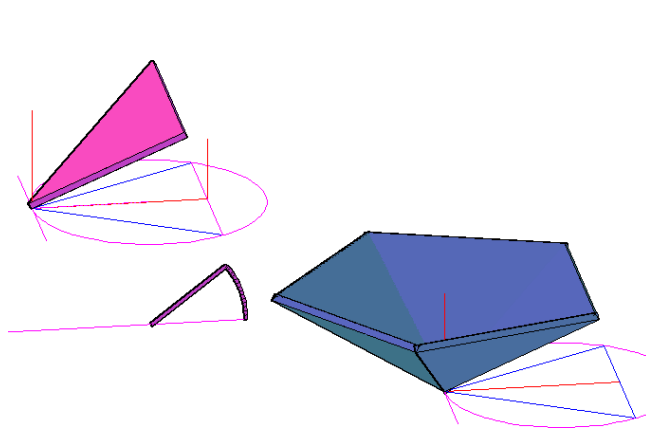

Figure 3a: Regular triangular 3D face model (the face primitive is understood as a triangular prism with a little height) of the icosahedron face rotated around the line parallel to the base of a regular triangle through the angle β and arrayed about the line perpendicular to horizontal projection (AutoCAD's 3D ARRAY command)
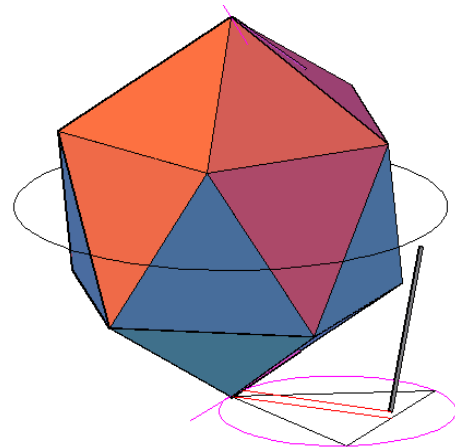

Figure 3b: The other faces adjacent to the faces of the lower part of the pentagonal corner are obtained by reflection (AutoCAD's 3D MIRROR command) with respect to a plane passing through the corresponding edge of the face and the center of the main diagonal of the icosahedron
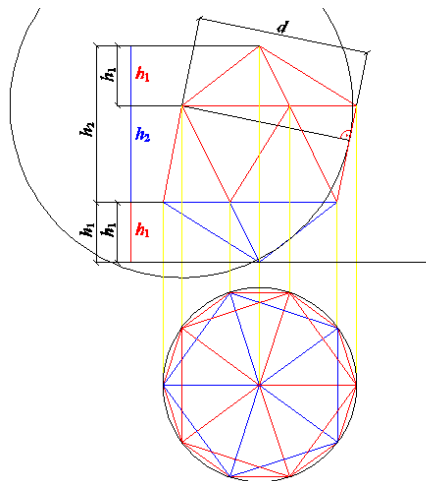
Figure 4a: The horizontal and vertical projection of a icosahedron and the manner of determination of the distance between the pair of two parallel faces
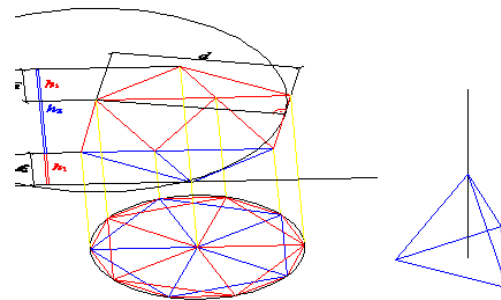
Figure 4b: The half of the segment line perpendicular to the pair of two parallel faces is the height of a regular pyramid with the regular triangular base

It is different, when we want to create a three-dimensional object in a CAD environment, eg. in the standard AutoCAD software. Among the Platonic solids only cube is available in the set of primitives of this software. All others, such as the height of a tetrahedron, octahedron diagonal, are created by setting previously geometrically necessary parameters. The construction of more complex Platonic solid models will be discussed in this paper as a proposal for teaching classes in Descriptive Geometry. This is, in a sense, a continuation of the other papers of the author of a similar nature [2, 3].

Following the Monge's projection of dodecahedron and icosahedron, well-known in Descriptive Geometry [1], we create their face and solid models in AutoCAD software in any version, e.g. AutoCAD 2013).

Having the Monge views of icosahedron with the main diagonal perpendicular to the horizontal projection plane (Fig. 2) we can easily construct the angles the edge (α) and face (β) form with the horizontal projection plane and the length of the main diagonal. These parameters are sufficient to construct a face model (Fig. 3ab). The face - a basic primitive in this construction - we model as a right triangular prism with a small height. For the construction of a monolithic model is needed the distance between arbitrary parallel faces of the icosahedron (Fig. 4a). Half of this distance is the length of a segment line, which is the height of the pyramid – the primitive in the construction. The base of the pyramid is a face of the icosahedron - an equilateral triangle. Further steps in AutoCAD software are transformations: planar symmetry (3D MIRROR), rotation (3D ROTATE), polar array (3D ARRAY) and Boolean operations (Fig. 4, 5). In a similar way we construct virtual models of the dodecahedron (Fig. 6, 7).
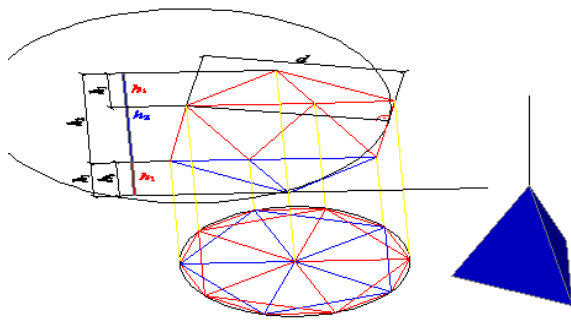
Figure 4c: 3D model of a regular pyramid of height equal to half of the height of the icosahedron with the regular triangular base as the face of this icosahedron – the primitive of a solid (obtained by AutoCAD's PYRAMID command)
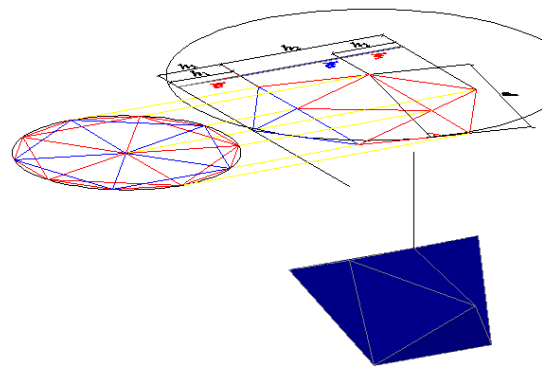
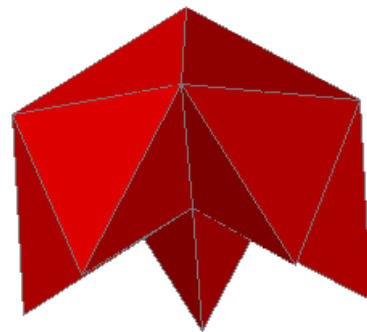Figure 4d: The creation of the 3D model of an icosahedron obtained by AutoCAD's 3D MIRROR command
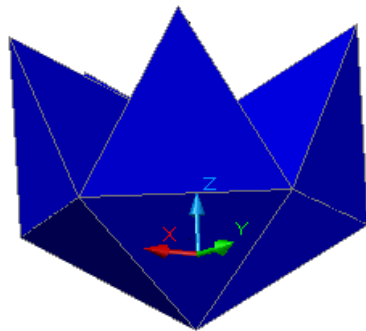

Figure 5a: Two solids composed with ten primitives transformed using AutoCAD's 3D MIRROR or/and 3D ARRAY commands
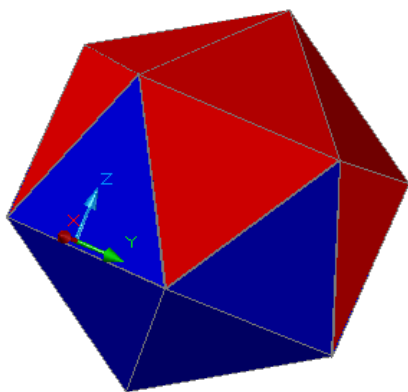

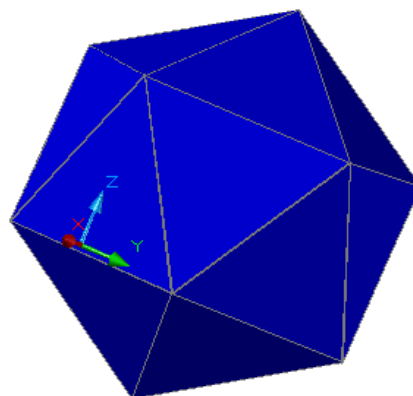Figure 5b: 3D model of the target solid consisting of two parts

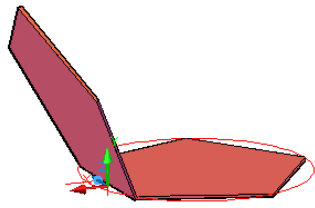Figure 5c: The final 3D model of the monolithic icosahedron

Figure 6a: Regular pentagonal 3D face model (the face primitive as a pentagonal prism with a little height) of the dodecahedron face rotated around the side of a pentagon (AutoCAD's 3D ROTATE command)
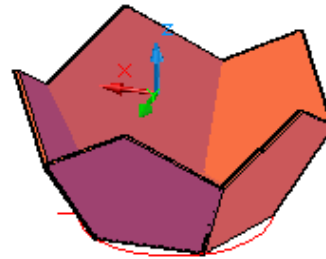

Figure 6b: The solid composed with six primitives transformed using AutoCAD's 3D MIRROR or/and 3D ARRAY commands
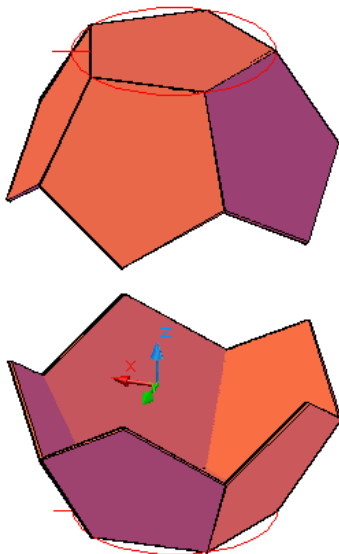

Figure 6c: Two parts of the created dodecahedron composed with six primitives transformed using AutoCAD's 3D MIRROR or/and 3D ARRAY commands
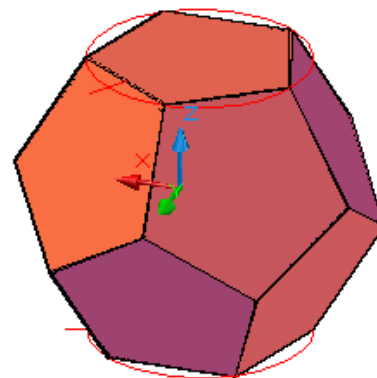

Figure 6d: The final 3D face model of the created dodecahedron
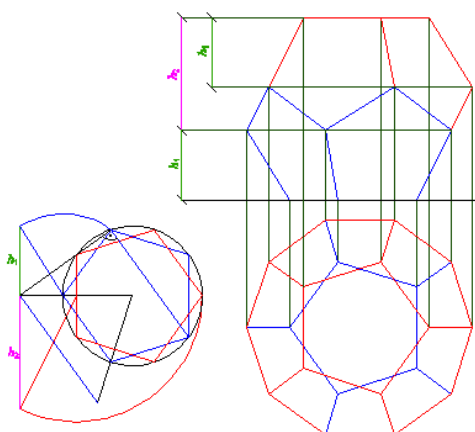

Figure 7a: The horizontal and vertical projection of a dodecahedron and the manner of determination of the distance between the pair of two parallel faces
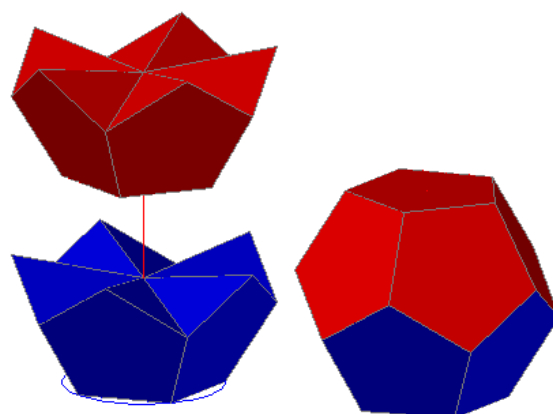

Figure 7b: The final 3D model of the monolithic icosahedron

3   Conclusion

As it seems, the proposal is a good justification of the need for the classical course of the Monge method, yet maintaining usability of the material. We should teach Descriptive Geometry in the context of spatial modelling. Such an approach, on the one hand will develop spatial imagination and, at the same time, motivate students to study the Monge projection method.

References:

[1]   Grochowski B.: *Geometria wykreślna z perspektywą stosowaną* (Descriptive Geometry and Applied Perspective (in Polish)). Wydawnictwo Naukowe PWN, Warszawa, 2010.

[2]   Koźniewski E., Orłowski M.: *From 2D Mongean Projection to 3D Model in AutoCAD.* The Journal Biuletyn of Polish Society for Geometry and Engineering Graphics, Vol. 22 (2011), 49-53.

[3]   Koźniewski E.: A Proposal for *Descriptive* Geometry Term Papers. The Journal Biuletyn of Polish Society for Geometry and Engineering Graphics, Vol. 26 (2014), 31-38.

[4]   Kurland K. S.: *AutoCAD 3D.Training Manual.* ® AutoCAD is a registered trademark of Autodesk, Inc. 2004 (pdf version)
http://www2.hcmuaf.edu.vn/data/dh04cc/file/Autocad%203d%20training%20manual.PDF

[5]   Stachel H.: *Why shall we also teach the theory behind Engineering Graphics.* Institut für Geometrie, TU Wien, Technical Report 35 (1996).

[6]   Stachel H.: *What is Descriptive Geometry for?* Dresden, Germany, February 27-March 1, 2003.

METODA MONGE'A W TWORZENIU MODELI WIELOŚCIANÓW FOREMNYCH

Fizyczne ściankowe (np. papierowe) modele wielościanów, w szczególności wielościanów foremnych tworzy się rysując wcześniej na płaszczyźnie ich siatki. Inne wymagania konstrukcyjne stawia nam zadanie wykonania monolitycznego (bryłowego) modelu materialnego za pośrednictwem np. modelu wirtualnego. Obok ścian bryły musimy znać wtedy ich wzajemne kąty nachylenia lub kąty nachylenia do wybranej płaszczyzny lub inne parametry takie jak odległość ścian równoległych. Jeśli wybierzemy drogę geometryczną, to bez wykonywania obliczeń można zrealizować to za pomocą programu AutoCAD. Wówczas bardzo pomocną okazuje się metoda Monge'a. Praca zawiera propozycję zrealizowania takiego zadania.