

MODELING AN EMBANKMENT WITH A NATURAL SLOPE

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Abstract: The paper presents a geometric characterization of models of embankments with a natural slope and discusses the relationship of these models with roof skeletons (straight skeletons), Voronoi diagrams for polygons (medial axis) and offset curves. Authors show AutoCAD commands, which can be used to generate geometric models of the embankments of any base.

Keywords: skeleton of roof, straight skeleton, circular cone, parabola, hyperbola, Dandelin's theorem, Voronoi diagram for polygon, medial axis, offset curve, internal friction angle, embankment

The paper is dedicated to the memory of Professor Bogusław Grochowski (1923-2011) in his 90th birthday anniversary.

1 Introduction

Among the geometric methods used in science, technology and many other areas an important role is performed by the theory of straight skeletons [1, 2]. In terms of three-dimensional, but also plane geometry, this theory is known as the geometry of the skeletons of roofs or briefly as a roof geometry [3, 8, 11,12, 14, 18]. This theory is in close kinship with Voronoi diagrams for polygon [14]. Both the straight skeletons and Voronoi diagrams for polygons are many interesting practical applications: the (semi) automatic reconstruction of urban models ([6, 7, 9,18, 22] and roads [3, 23] based on satellite images, in cartography and photogrammetry [10], morphology analysis of the grain structure of the material [21], in medicine for representation, reconstruction and visualization of human organs [5], in the design of earthwork organization [13], to name just a few. In the present work we use the geometry of the roof to the geometric modeling of embankments on predetermined base line and a natural slope.

2 Create real and virtual models of the natural embankment

The starting point for consideration is, in a sense, a simple experiment. In the laboratory, we cut a cardboard polygon, which can be considered as a base model of the roof (of a special polyhedral surface) [11, 12, 17]. We then place a cardboard at a certain height over a flat surface polygon. Next, any granular material is poured over the cardboard polygon (Photo 1). The mechanics of granular materials indicates that a material sprinkled in one spot forms a heap, which takes the shape of a cone. The angle at the base of the cone is called the *internal friction angle* [20]. This angle depends on the type of material and is an important parameter which characterizes the mechanical properties the material. In this case the material is placed in a embankment, which is similar to the roof with the constant slope [14,17]. But there is a

difference, because in the neighborhood of the vertex concave angle of the base a curved surface is created and the ridge line of the roof is a curved line.



Photo 1: A heap of granular material (the polygonal cardboard base with two concave angles)

By preserving the angle of internal friction, the ground under the force of gravity slides in perpendicular direction to the edge of the base. Let's create a geometric model of the embankment.

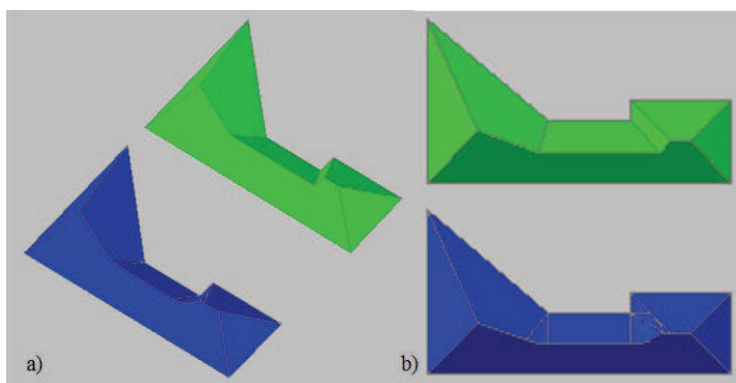


Figure 1: a). The virtual model of granular material embankment (at the bottom), at the top - the model of a roof with the same base (made in AutoCAD program); b) The horizontal projection of a virtual model of a heap of granular material (at the bottom), at the top - the model of a roof with the same base (made in AutoCAD program)

Straight lines converging at the vertices of concave angles, form a right circular cone. Simple reasoning shows that some parts of the ridge line are parabolas. Indeed, the generatrices of a cone with the apex angle of $90 - \varphi$ at the vertex of a connected concave base form an angle φ with the plane of the base. Similarly, all the plane faces of the model of a heap form an φ angle with a plane base. Therefore, each face is parallel to one (and only one) *generatrix* of a cone and according to the Dandelin's theorem it cuts the cone surface in the form of a parabola [4]. Figures 1 a) and b) show geometrical models of piles of granular material obtained in the experiment (made in the CAD environment).

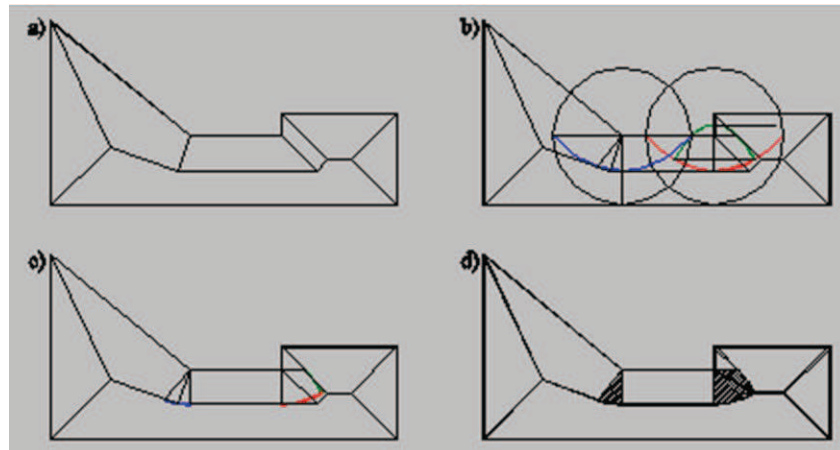


Figure 2: Roof skeleton and Voronoi diagram (medial axis): a) horizontal projection of a roof; b) transformation of a roof to Voronoi diagram for a polygon; c) selected sections of the parabolas; d) Voronoi diagram for a polygonal base with the hatched parabolic areas

The horizontal projections of a roof model and a model of granular material embankment are well-known geometric objects: *straight skeleton (roof skeleton or briefly a roof)* and *Voronoi diagram for a polygon* [13,14, 19].

In the case of the more complex polygon-shaped bases, it may happen that the concave corners of the polygon will be opposite each other, in a sufficiently small neighborhood. Then the cones induced by the vertices of the angles, overlapping each other along the hyperbola, will create a common embankment ridge line (Fig. 2). The top view of the common curve of two cones is a segment of a straight line (Fig. 5b, 4).

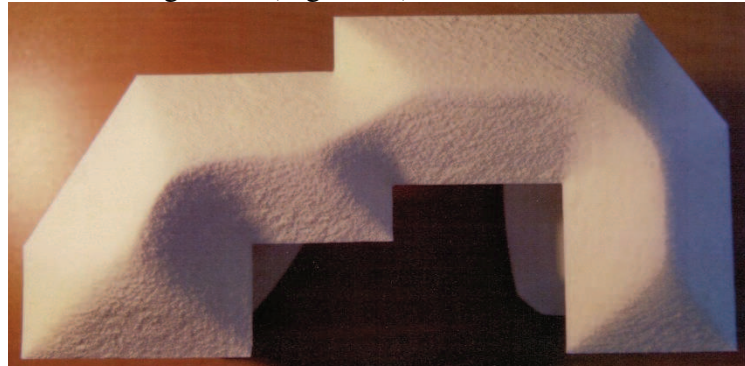


Photo 2: A heap of granular material (the polygonal cardboard base with four concave angles), including angles with two opposite concave in a sufficiently small neighborhood

The resulting 3D modeling through the roof (Fig. 4) does not reflect the prism obtained through the free formation of a flat base on granular material (Fig. 1b). So it is necessary to "correct" the surface of the model and its ridge line. The walls of the embankment model, as we previously noted, may be fragments of the planes or the surface of the circular cone. Thus, the ridge line will consist of line segments (where two planes meet), parabolic arcs (where a plane intersects the surface of the cone), hyperbolic arcs (where two conical surfaces meet).

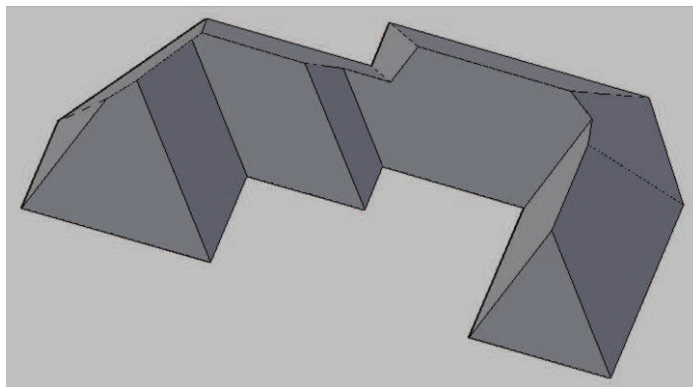


Figure 3: The 3D model of a roof with the same base as in Figure 4 (made in AutoCAD environment (EXTRUDE and TAPER commands with the appropriate angle))

And so, in Figure 4 the ridge line embankment model consists of straight lines: AB , BC , EF , JL , OP ; parabolic arcs: CD , DE , FH , IJ , LM , MN , NO (FG , JK – parabolic arcs as a part of the edge of the corner); hyperbolic arc: HI . Figure 4 shows the medial axis of the polygon.

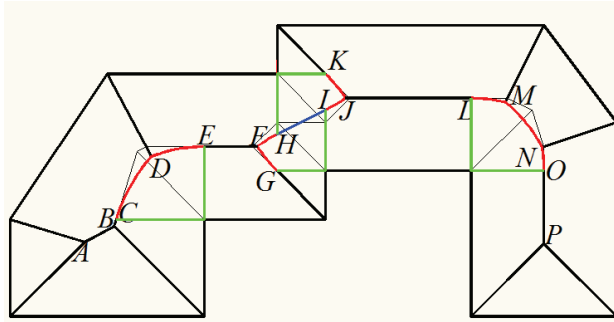


Figure 4: The top view of the ridge line of the roof prism model made on the polygon with two opposite concave angles consisting of: the straight line segments: AB , BC , EF , JL , OP ; the parabolic arcs: CD , DE , FH , IJ , LM , MN , NO (FG , JK – arcs of parabola as parts of corner ridges); the straight line segment (hyperbolic arc in 3D) HI

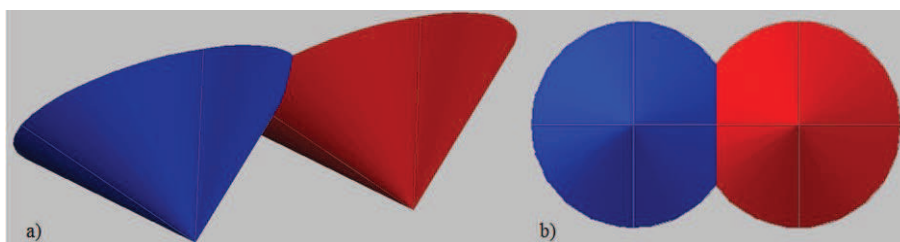


Figure 5: a) A virtual model of the common curve (hyperbola) of the two surfaces of circular cones made in the AutoCAD environment - isometric view; b) a virtual model of the common curve (hyperbola) of the two surfaces of circular cones made in the AutoCAD environment – top view (the hyperbola is seen as a straight line)

3 3D model of the embankment with natural slope on the plane in the advance given region

Suppose now that there is any area of the plane, the boundary of which is any closed planar curve (Fig. 6a). Let us formulate the task: find a surface of constant slope, passing through the boundary curve. We are dealing with the so-called constant slope surface.

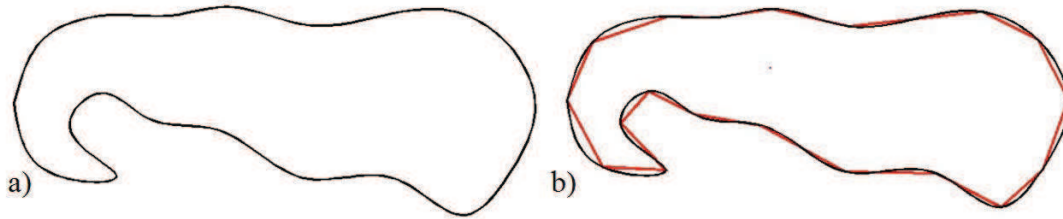


Figure 6: a) Any planar area on a regular shoreline (realized as a spline line in AutoCAD); b) a closed broken line inscribed in the curve as the base of a roof

Let an area of the plane, with the boundary line as a closed curve be given (Fig. 6a). Let us inscribe any broken line in this curve (Fig. 6b). We obtain, the polygon inscribed in the curve (Fig. 6b).

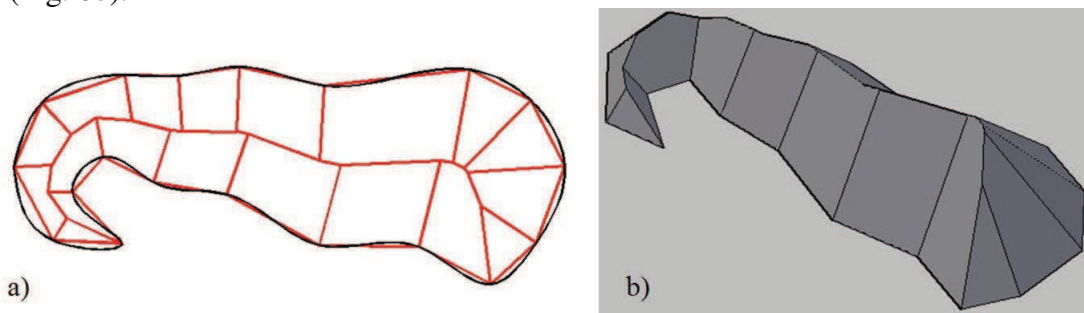


Figure 7: a) model of a roof spanned on a polygon inscribed in the boundary curve of the area - the top view; b) model of a roof spanned on the polygon inscribed in the boundary curve of the area – implementation in AutoCAD (commands: EXTRUDE and TAPER)

This polygon is treated as the basis for the roof [11] (straight skeleton [1]). Then, we construct a straight skeleton (Fig. 7a) and in fact the roof (Fig. 7b), which is the surface of equal face inclination [14]. Then the approximation of the required embankment surface model is the question of desired accuracy (Fig. 8a). If you are interested in a particular model of the embankment, it is sufficient to cut off part of the created body (Fig. 8b).

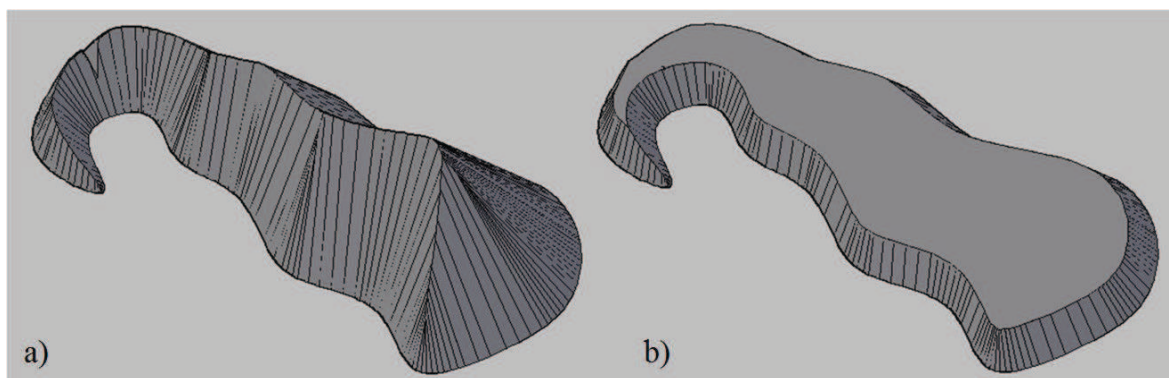


Figure 8: a) Model of the embankment with a natural slope (the inclination angle equal to the angle of internal friction) spanned on a polygon inscribed in the boundary curve - broken curved edge approximation with greater accuracy – implementation in AutoCAD (commands: EXTRUDE and TAPER used to the polyline or spline); b) model of the embankment with a natural slope and given height – implementation in AutoCAD (Boolean operations on created in Figure 8a solid)

Let's go back to the laboratory again. Let us cut this time, the area (for example, a hard cardboard, respectively) such as shown in Figure 6a. We then place a cardboard at a certain height over a flat surface polygon. Next, any granular material is poured over the curvilinear cardboard region (Photo 3). We get a real model for the embankment of the natural angle of internal friction of the material.

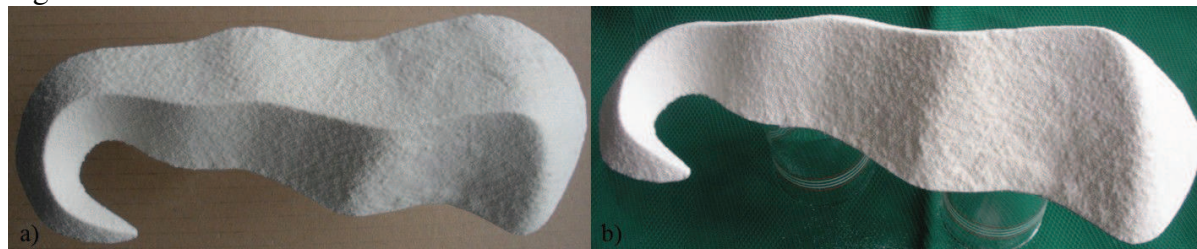


Photo 3: a) A real heap of granular material (the curvilinear cardboard base – spline boundary curve from Figure 6a) – the top view; b) a real heap of granular material (the curvilinear cardboard base – spline boundary curve from Figure 6a) – the axonometric view

4 The offset curve approach to the construction of 3D model of the embankment with natural slope

Modeling of the embankment can be done by contour curves that are offset curves. Classical surfaces for which the slope of the projection plane at any point is constant are a circular cone and a plane. Contour lines of these surfaces are offset curves: circles and straight lines [15]. Note that the tangent plane to the topographic surface can be determined by two mutually perpendicular tangent straight lines, one of which is a horizontal line, the other is a line of slope. The projections of these straight lines, mutually perpendicular, are perpendicular under characteristic invariant of orthogonal projection and thus define the offset curve points (contour line). Therefore, in the case of a polygon with concave corners the offset curves beside line segments comprise arcs of circles with the centers at the vertices of concave angles (Fig. 9a). In 3D space such offset curves induce a circular cone (Fig. 9b). To realize the 3D model in AutoCAD (Figs 9 and 10) at the first we create in 2D appropriate contour lines as the offset curves. Next, we change the elevations of the 2D contour lines of the site to the appropriate heights. At the end we use the command LOFT [16].

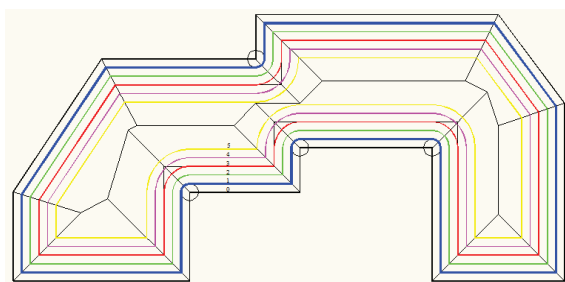


Figure 9: a) Geometric contour lines of the truncated solid model of the embankment with a natural slope from Photo 2 as the offset curves (line segments and arcs of circles) - implementation in AutoCAD (command: OFFSET)

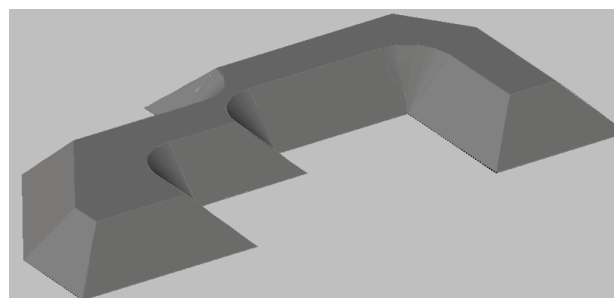


Figure 9: b) The truncated 3D solid model of the embankment with a natural slope from Photo 2 - implementation in AutoCAD (command: LOFT used to polylines)

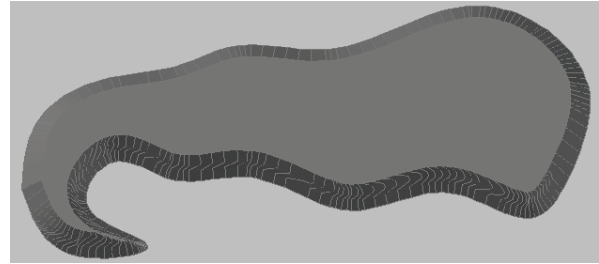
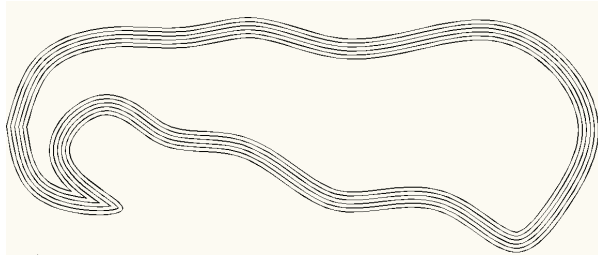


Figure 10: a) Geometric contour lines of the solid model of the embankment with a natural slope from Photo 3 as the offset curves (line segments and arcs of circles) - implementation in AutoCAD (command: OFFSET)

Figure 10: b) 3D solid model of the embankment with a natural slope from Photo 3 - implementation in AutoCAD (command: LOFT used to polylines - offset curves)

5 Conclusion

The method allows to create a natural slope embankment stretching over any area, the boundary of which is any closed planar curve. In connection with the implementation of algorithms for creating simple skeletons in AutoCAD, we can perform modeling of slope surfaces in CAD environments, which are used for design processes to topographic terrain. The optional application of offset curves is the essential method in the discussed method of modeling. So it is a universal method for designing any type earthwork.

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MODELOWANIE NASYPÓW O NATURALNYM NACHYLENIU

W pracy przedstawiono geometryczną charakteryzację modeli nasypów o naturalnym kącie nachylenia powierzchni stokowej tworzącej nasyp. Wskazano na ich powiązanie ze szkieletami dachów (prostymi szkieletami) i diagramami Voronoi dla wielokątów. Pokazano za pomocą jakich poleceń programu AutoCAD można realizować geometryczne modele nasypów o dowolnej podstawie.