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GEOMETRICAL ASPECTS OF RECONSTRUCTION OF THE CYLINDRICAL PANORAMA

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Abstract. The paper is a sequel to the author's previous study dealing with direct means of constructing cylindrical and conical panoramas, as well as possible application of computer software to their graphical mapping. This paper presents geometrical approaches in formation of the algorithm for making up the panorama of objects, when their spatial model is not available. This algorithm allows to draw a flat panoramic image of any straight line passing through two points given directly in the unrolled panorama background. It can be useful in restoration of the panorama paintings. The algorithm also calculates spatial coordinates of the characteristic points of the objects on the base of their panoramic image, which enables 3D reconstruction. The paper shows simple examples of application of the presented algorithm for reconstruction of fragments of panorama images. The algorithm is created in Mathcad program. However, it can be implemented in AutoCAD one or any graphical package. Presented study was motivated by the main question, how much geometrical relations and knowledge can help in some kind of reconstruction or 3D recovery from the single panorama image.

Keywords: panoramic projection, cylindrical perspective, reconstruction of panorama, computer aided drawing

1 Introduction

Investigation of a cylindrical panorama for using it in recovery of the model of the presented object is a very fast developing field recently. Panoramic images from multiplestations can be used for 3D reconstruction and documentation of the architectural objects [1], [2]. The task of retrieving information about a spatial structure of a figure from a single panoramic image is not easy. Due to the fact that, any construction and reconstruction of every panorama has to be submitted to geometrical rules regarding that kind of mapping, it is possible in some cases, however. The analysis of such rules and discussion of geometrical relations, which could help in some kind of 3D reconstruction and 2D recovery of the single panoramic image is the main aim of the author.

2 Basic information

In the author's previous works [3],[4],[5] the method of the direct construction of cylindrical and conical panoramas has been introduced. It was realized by means of two- projective partly composed representation. In general, that representation is composed of two projections: the main one and the supplemental one. It has the property that both; given figure and the image of that figure received earlier in the supplemental projection, are represented in the main projection. In the panorama representation the main projection is a central projection from the center S onto a cylindrical surface τ, whereas the supplemental projection is the orthogonal projection onto a base plane π . Consequently, in that method the panorama image of any

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point F is a pair $\{F^S, F^{O,S}\}$ of points, while the image of any straight line m not particularly situated towards an apparatus is composed of two curves of ellipses (Fig.1).

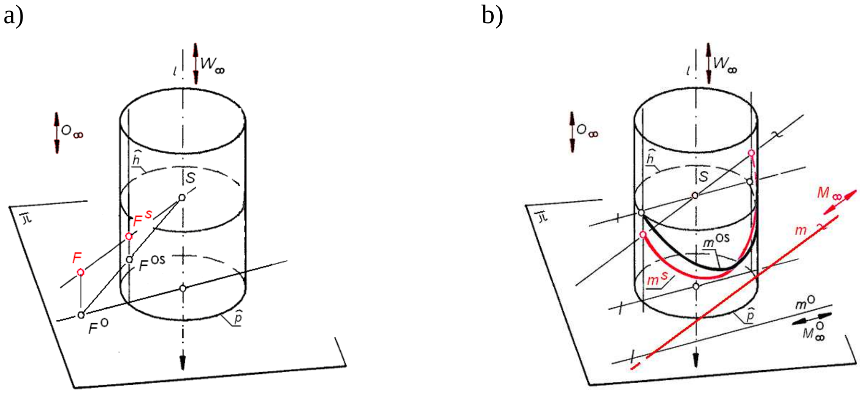

Figure1: The representation of the: a) point F, b) straight line m in the panorama projection

For the graphical mapping effects of the representation on the flat surface, images contained in the τ background are transform to their counterparts on an unrolled background. It can be realized by projecting the cylindrical background from the centre S onto the base plane π (Fig.2).Next it enables establishing projective relations between points on generatrices obtained in that projecting and their counterparts on the unrolled background.

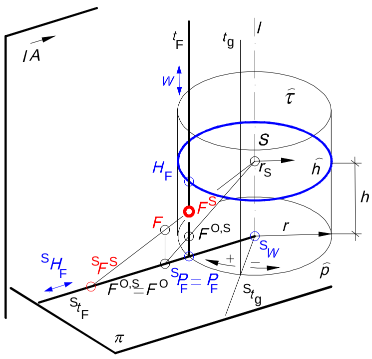

Figure 2: The projection of the straight line t_F with the series ${}_S R\ t_F(P_F, H_F, W, F^{\cdot S}, \ldots)$ of points for realization of the transformation

According to the figure 3, series of points on generatrix ${}^S t_F$ obtained in the central projection from the center S onto the base plane π and series of proper points on the straight line t_F^T (received as a result of the translation of the generatrix t_F^R given on the unrolled background) are projective ones.

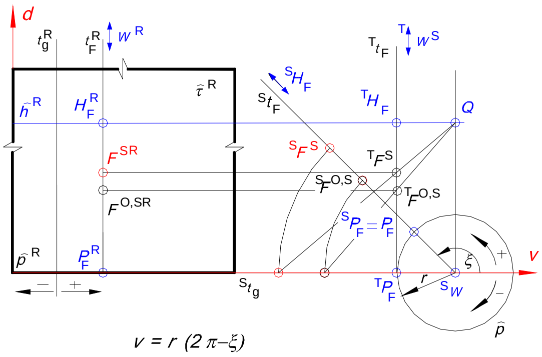

Figure 3: Graphical connections between the mapping F^{SR} of the point F on the unrolled background $\hat{\tau}$ and its mapping ${}^S F^S$ received by the projection from the center S onto the plane π

Above graphical relations have helped the author to create the algorithm for drawing the cylindrical panorama of the given model objects, which has been implemented in the previous work [4]. In this paper they help to create the algorithm for making up the panorama image of objects when their spatial model is not available. They make it also possible to calculate spatial coordinates of the characteristic points of objects on the base of their panoramic image, which can enable 3D reconstruction.

3 Mapping the straight line going through two points given on the unrolled background of the cylindrical panorama

The projective relation between points included in any generatrix $t_F^R \ni F$ on the unrolled background of the cylindrical panorama, and the proper points on the generatrix ${}^S t_F$ obtained in the central projection from the centre S onto the plane π means equality of the relationships of the double division of homologous fours of points contained appropriately in generatrices t_F^R and ${}^S t_F$ (Fig.3).

For homologous fours of points $\{{}^S W, {}^S F^{O,S}, {}^S P_F, {}^S H_F\}$ and $\{W^R, F^{O,SR}, P_F^R, H_F^R\}$ contained in series $<\emptyset, {}^S t_F>$ and $<\emptyset, t_F^R>$ above relation can be written by the formula below:

$$\frac{\left|{}^S F^{O,S}\,{}^S P_F\right|}{\left|{}^S W\,{}^S P_F\right|} : \frac{\left|{}^S F^{O,S}\,{}^S H_F\right|}{\left|{}^S W\ {}^S H_F\right|} = \frac{\left|F^{O,SR}\ \ P_F^R\right|}{\left|W^R P_F^R\right|} : \frac{\left|F^{O,SR} H_F^R\right|}{\left|W^R H_F^R\right|} \tag{1}$$

In a row series of points contained in the straight line m and series of points contained in the generatrix $t_F \in \hat{\tau}$ are projective ones (Fig.4). Moreover, the series $<\emptyset, t_F^S> = t_F\,(W_\infty^S, P_F^S, H_F^S, \ldots)$ and $<\emptyset, t_F^R> = t_F^R\,(W_\infty^R, P_F^R, H_F^R, \ldots)$ are congruent.

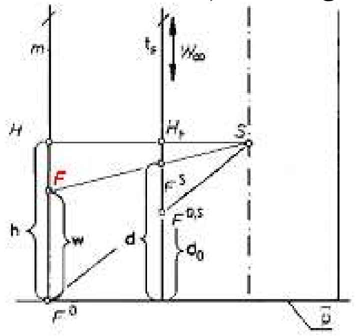

Figure 4: The perspective relation between points contained in the straight lines m and t_F

It enables establishing the relation below:

$$\frac{\left|F^O H\right|}{\left|F\ H\right|} : \frac{\left|F^O W_\infty\right|}{\left|F\ W_\infty\right|} = \frac{\left|F^{O,S} H_F\right|}{\left|F^S H_F\right|} : \frac{\left|F^{O,S} W_\infty\right|}{\left|F^S W_\infty\right|} \tag{2}$$

Projective relations mentioned earlier help to establish the distance d as the function of the length v of the unrolled panorama background for the angle ξ (Fig.5a). We omit describing our algorithm in detail, but show the work of it after its implementation in Mathcad program.

The computed algorithm enables drawing a panoramic image of any straight line passing through two points, given in the unrolled panorama background. The points can be establish directly in the unrolled background of the panorama. The program displays their coordinates, and after putting them as variables of the created function, it draws the line as a plot (Fig.5a,5b). The figure number 5a shows the example of drawing the line passing through two given points, with following coordinates: (80,3), (110,9), whereas the figure number 5b shows the example of making up the line going through two corners of the model building.

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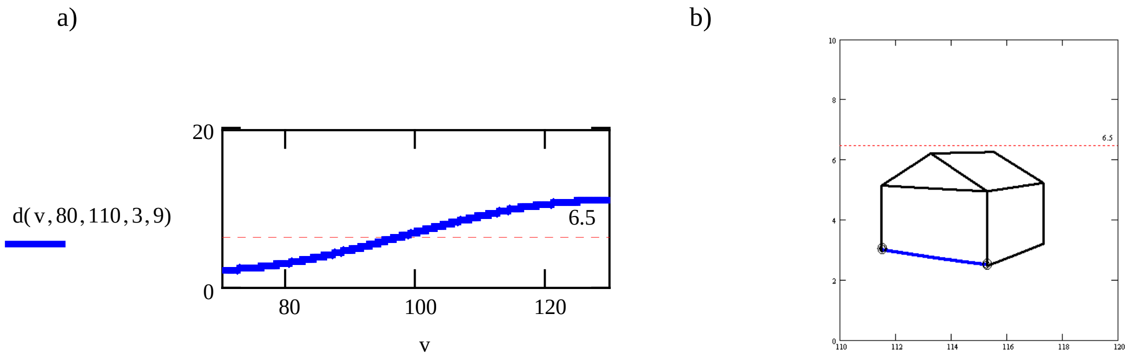

Figure 5: a) The panoramic mapping of the straight line in Mathcad 2000 Professional program, b) The example of making up missing line of the panoramic image

4 The use of the algorithm for 2D reconstruction of the panorama image

Before any reconstruction can be done some prior knowledge about geometrical properties, parallelism, or shape of the shown object is necessary. Then, the presented algorithm can be apply to complete missing parts of the panorama image. For this purpose it is first necessary to establish height *h* of the horizon line, as well as length *r* of the radius of the base circle. In case of the historical panorama painting length *r* can be calculated by the program on the base of the length of the canvas, whereas height *h* of the horizon line is established on the base of the vanishing point of images of any pair of parallel straight lines of the objects. Next, our algorithm can be used to draw a draft of missing lines of the panorama (Fig.6). In case of the lines which are not straight lines in reality, it is recommend to divide them into some segments of lines, which can be approximated by the straight lines.

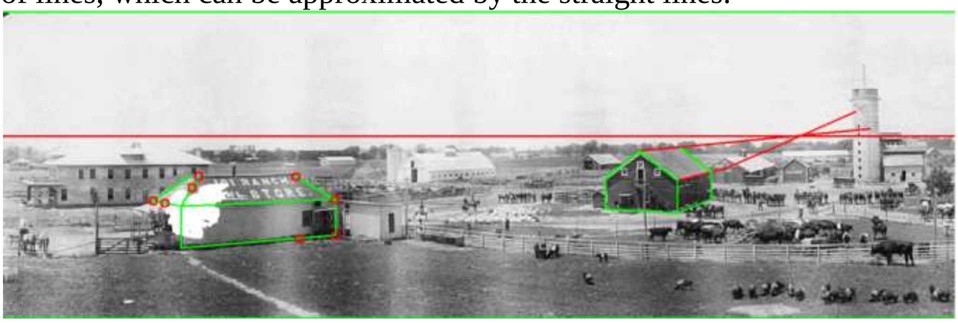

Figure 6: The application of the algorithm to 2D reconstruction of the panorama image of the historic *101 in Ranch Oklahoma* [7]

The algorithm was computed in Mathcad 2000 Professional program but it can be implemented in any graphical package to make drawing panoramas more efficient.

5 The use of the algorithm for 3D reconstruction of the panorama

The task of retrieving 3D information from a single panorama image is not always easy. However, after some transformations our algorithm can be useful for creating the 3D edge model of the panorama image. For 3D reconstruction of the invisible lines the application of the concept of vanishing points is used. The program calculates vanishing points for parallel lines and draw not visible lines.

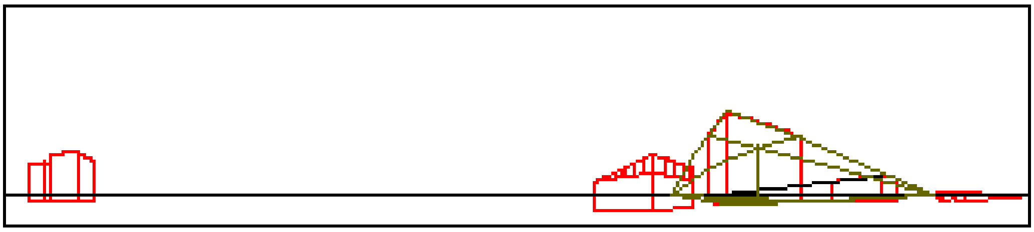

Figure 7: Establishing vanishing points for parallel straight lines on a panoramic image

The first step of the reconstruction of it is finding height *h* of the horizon line, as well as length *r* of the radius of the base circle (Fig.7). Next, several invisible lines of the panorama image are made up. After making up segments of lines included in the base plane, it is possible to prepare the ground map of the objects (Fig.8a). Next, the spatial coordinates of the characteristic points of the objects are calculated by the program. They establish the edges of the presented panorama objects. Finally, the created edge model of the displayed objects can be a basis of the subsequent visualization (Fig.8b). Calculation of the coordinates of the characteristic points are prepared in Mathcad program, whereas the special edge model is created in AutoCAD one.

a) b)

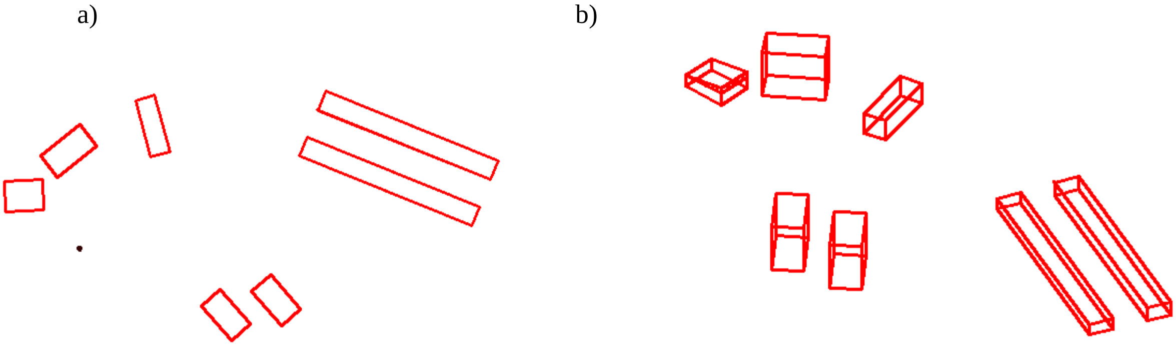

Figure 8: Model objects: a) the ground map of them, b) the 3D model of them

As the example of the application of the algorithm 3D reconstruction of the fragment of the panorama painting is presented (Fig.9a,9b).

a) b)

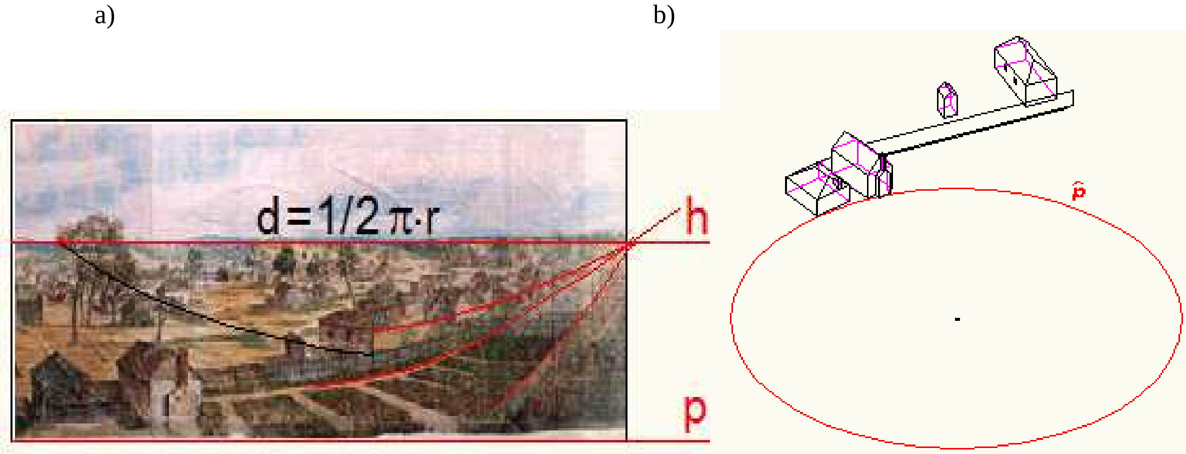

Figure 9: a) Given fragment of panorama image[4], b) The 3D edge model of the panorama

Conclusions

The study was motivated by the general question, how much geometrical relations can help in some kind of reconstruction or 3D recovery from the single panoramic image. Due to the fact that, panoramic paintings were prepared with a great precision, every panoramic representation is submitted to geometrical rules regarding that kind of mapping. It enables finding graphical connection between the object and the flat panoramic image of it, displayed in the unrolled background. Next, it is useful for establishing an analytical algorithm for drawing missing lines of the unrolled panorama, as well as in preparing 3D model of it. 3D reconstruction on the base of the single panoramic image is possible, however, only in case, when the shape and the ground map of the presented objects can be establish. The presented examples of the application of the algorithm, although simple ones, permit to state it works well in Mathcad program. However, it can be implemented in any graphical package to make reconstruction more efficient.

References:

[1] Luhmann T.: *Panorama Photogrammetry for Architectural Applications*. Mapping, ISSN 1131-9100, Nº 139, 40-45, 2010.

[2] Luhmann T., Tecklemburg W.: *3-D Object Reconstruction from Multiple-station Panorama Imaginery*. International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences, Vol. XXXIV, Part 5/W16, H.-G. Maas and D. Schneider.

[3] Dźwierzyńska J.: *Zastosowanie Dwurzutowych Odwzorowań Częściowo-Złożeniowych do Bezpośrednich Konstrukcji Rozwinięć Panoram Walcowych i Stożkowych przestrzeni E_3.* Praca doktorska, Politechnika Rzeszowska, 2005.

[4] Dźwierzyńska, J.: *Cylindrical Panorama-Two Approaches*. The Journal Biuletyn of Polish Society for Geometry and Engineering Graphics, Vol. 19, 9-14, 2009.

[5] Dźwierzyńska J.: *Geometrical Aspects in the Reconstruction of the Cylindrical Panoramas*. Proceedings of 14th International Conference on Geometry and Graphics, Kyoto, Japan, 2010.

[6] www.panoramapainting.com

[7] http://upload.wikimedia.org/wikipedia/ commons/2/28/RanchOklahoma.ws.jpg

GEOMETRYCZNE ASPEKTY REKONSTRUKCJI PANORAMY CYLINDRYCZNEJ

Artykuł jest nawiązaniem do wcześniejszych rozważań autora dotyczących konstrukcji panoram walcowych. Przedstawia geometryczne podejście przy tworzeniu algorytmów pozwalających na komputerowe wspomaganie przy rekonstrukcji 3D obiektu przedstawionego na obrazie panoramicznym, którego model przestrzenny nie jest dany. Algorytmy tworzy się w programie Mathcad, a następnie mogą być one zastosowane w programie AutoCAD lub dowolnym programie graficznym, co zwiększa efektywność rekonstrukcji. W artykule pokazuje się przykłady zastosowania algorytmu.