OPTIMISATION OF WINDOWS’ GEOMETRIC PROPERTIES IN THE ASPECT OF BUILDINGS ENERGETIC PROTECTION DESIGN

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Abstract Heat wastes by the windows are crucial factors from the point of view of energy conservation in building design. In the article a problem of window shape and size optimization was discussed, connected with:
- assessment of values being the function of natural lighting,
- building insulation.
The aim of the research carried out by the author is the improvement of methods accepted as standards in Ukraine.

Key Words: natural illumination, insolation, solar map.

1 Introduction
The minimal size of a window is calculated basing on the recommended relations of the area of window to the area of floor. The minimal area of windows is calculated in most countries by using the daylighting factor (DF)

\[ e = \frac{E_{\text{int}}}{E_{\text{ext}}} \times 100 \] (in percent),

in which \( E_{\text{int}} \) is the natural light exposure of the design point on the working plane in the room, and \( E_{\text{ext}} \) is the outdoor horizontal light exposure under the open sky.

The standard values of DF are defined for the critical external light exposure (5000 luxes in Europe) from the condition of sufficiency of light exposure of the design point for performance of a prospective kind of visual work.

In the case of cloudy sky, the DF is calculated by using various analytical and grapho-analytical methods [1]. In Ukraine and some other countries the DF is calculated in two stages.

- at the first stage, the geometrical DF (GDF), denoted by \( \varepsilon \), is calculated by (1) under the following restrictions:
  (i) brightness of the sky is identical for all directions.
  (ii) the window is an aperture in a wall or covering, without glazing and frames.
  (iii) all light exposure is caused by skylight and reflections are not considered,
- at the second stage, GDF is multiplied by some factors that take of restrictions (i)-(iii).

The GDF is calculated by using the radial projection of a window to the sphere of unit radius (whose centre coincides with the design point) with the subsequent re-projection of the obtained compartment of sphere to the working plane by beams that are perpendicular to it. In this case we have

\[ \varepsilon = \frac{\sigma}{\pi} \times 100. \] (2)

In Ukraine, the grapho-analytical calculation of GDF is realized by the method of beams. In this method the equi-bright hemisphere of the sky is broken by two pencils of beams.
planes into 10000 fragments whose projections to the horizontal plane are equal. The first pencil consists of 100 planes that pass through some diameter of the base of hemisphere, and the second consists of 100 planes that are perpendicular to this diameter. If the number $N$ of fragments of the sky hemisphere that are visible from the design point is known, then equation (2) can be rewritten in the form

$$\varepsilon = 0,01N.$$  \hspace{1cm} (3)

The number $N$ can be found by Graphs I and II. The beams of Graph I are the lines of crossing the planes of the first pencil with the plane that passes through the centre of hemisphere perpendicularly to diameter. The beams of Graph II are the lines of crossing of any plane in the first pencil with the planes of the second pencil.

The standard design procedure of DF [2] does not use all resources of the method of beams. For example, it does not use the method of calculating DF for light openings that are partially eclipsed by surrounding building. It does not use the method of calculating DF for inclined windows, neither does it consider the light arriving from the earth surface to the design points on inclined surfaces. Section 2 shows how to correct this.

The second important function of windows is the insolation of housings. In Ukraine the duration of insolation is normalized. It is considered equal to the duration of insolation of some design point, usually the window centre. This method allows finding the time of insolation only for windows whose contour of external aperture is similar to the contour of glazing, and only if both contours are flat, convex, and central-symmetric. For other forms of windows it is possible that the design point is not insolated but solar beams get into a room. To find insolation in this case, we propose to use the method of boundary surface insolation, which is considered in Section 3.

2 Natural illumination

The following formulas can be used for calculating DF:

For lateral illumination:

$$e = \left(\sum_{i=1}^{m_i} \varepsilon_{si} q_i + \sum_{j=1}^{m_j} \varepsilon_{bj} R_j + \sum_{k=1}^{m_k} \varepsilon_{sk} A_k\right) r_1 \frac{\tau_E}{K_f};$$  \hspace{1cm} (4)

For upper illumination:

$$e = \left[\frac{\sum_{j=1}^{N} \varepsilon_{ij}}{N} r_2 (K_t - 1) - \tau_E\right] \frac{\tau_E}{K_f};$$  \hspace{1cm} (5)

$$e = \sum_{i=1}^{m_i} \varepsilon_{si} q_i + \sum_{j=1}^{m_j} \varepsilon_{bj} R_j,$$

Here $\varepsilon_{si}, \varepsilon_{bj}, \varepsilon_{sk}$ are GDF in design point; they make allowance to direct light from $i$-th site of the sky, to light reflected from the $j$-th opposite house, and to light reflected from the $k$-th fragment of the ground surface; $q_i$ makes allowance to the brightness of $i$-th fragment of the sky; $R_j$ makes allowance to relative brightness of $j$-th opposite house; $A_k$ is the albedo of $k$-th fragment of the ground; $m_i, m_j, m_k$ are the numbers of fragments of the sky, of opposite houses, and of fragments of the ground that are visible from the window in design point; $r_1, r_2$ makes allowance to diffuse light reflected from surfaces of premises at lateral and top illumination, respectively; $\tau_E$ is the common factor of optical transmission of windows; $K_f$ makes allowance to cleanliness of the window; $K_l$ makes allowance to type of lantern of ceiling light;
N is the number of design points on characteristic cut of a house (at least 5). The values of factors $R_i$, $q$, $r_1$, $r_2$, $\tau$, $K_f$ and $K_l$ are found by [2].

The design value of DF in each design point is defined as the total value of DF at this point from all windows.

Figure 1 illustrates the finding of $\varepsilon_s$ for a window of any form not shaded by surrounding buildings in design point lying on a plane of general position.

First of all, we find the geometrical centre $D$ of the part of window located above the plane of the horizon.

Then we find the line $a$ of crossing the plane of window with the working plane. Through the design point $A$, we draw the plane $\alpha = ABC$ that is perpendicular to $a$, in which $C$ is the orthogonal projection of $D$ to $\alpha$.

The part of window located above the plane of horizon is replaced by a rectangle $1234$ of the same area that is constructed as follows. We inscribe it into a rectangle $1'2'3'4'$ with sides $1'4'$ and $2'3'$ that are parallel to $a$. Define the area $F_{1'2'3'4'} = l' \cdot m'$ of rectangle $1'2'3'4'$ and area $F_s$ of the considered part of window. Calculate $k = \sqrt{F_s/F_{1'2'3'4'}}$. Accepting the point $D$ as the centre of symmetry, we build a rectangle $1234$ with sides $|12| = |34| = l = k \cdot l'$; $|23| = |14| = m = k \cdot m'$; the sides of $1234$ must be parallel to the corresponding sides of $1'2'3'4'$.

Further, we superpose Graph I with the plane $ABC$ in such a way that the base of the graph coincides with the line $AB$, and its pole $O$ coincides with $A$. Calculate the number of beams $n_1$ passing through the window $1234$ to the point $A$ in Graph I.

We superpose the Graph II with the plane $ACD$ such that the pole $O$ of the graph superposes with $A$, and its base with the line of crossing of the working plane with the plane $ACD$. Calculate the number of beams $n_2$ that pass through the window $1234$ to $A$. The GDF from the celestial sphere $\varepsilon_s$ is defined by (3) in which $N = n_1 \cdot n_2$.

Similarly, we find the value $\varepsilon_e$ from the part of window under the plane of the horizon. If the window is shaded by the neighboring houses, then the part of window $w_{bj}$ that is shaded, its part $w_{sj}$ through which the firmament is observed, and its part $w_{ek}$ through which the ground is observed, are considered as separate windows.

This method for calculating $\varepsilon_s$, $\varepsilon_b$ and $\varepsilon_e$ is realized in MathCAD.

3 Insolation

It is convenient to calculate the time of insolation by using solar cards. The solar card is the stereographic projection from Nadira's point of the celestial hemisphere with the solar
trajectories put on it, to the base plane of the hemisphere. Usually the pole of solar card is superposed with the design point, and the window, shading elements of facade and opponent buildings are stereographically projected to the solar card [3]. This method is very illustrative and informative.

For calculating the duration of insolation of rooms with windows of complicated form, we propose to use the method of boundary surface. By the boundary surface of insolation we mean the surface of glass cover of windows. The room is insolated if the boundary surface of insolation is insolated. The method is based on a space transformation that transforms the boundary surface of insolation to the point at the centre of solar card.

Consider this transformation presented on Figure 2. Let the plane of glass cover be bounded by a curve \( w \), and solar beams get into the room through the area bounded by a curve \( s \). The horizontal area of insolation is filling by traces of all vertical planes that cross both focal curves. If we take from this two-parameter set any pencil of planes with a parallelism plane \( \alpha \), then we can define for this bunch a pair of crossed straight lines \( W_1S_1 \) and \( W_2S_2 \) that have the maximal corner of crossing \( \beta_{\alpha} \).

Let a curve \( w \) of space \( \Phi \) correspond to a point \( \overline{W} \) of the space \( \overline{\Phi} \). Draw from the point \( \overline{W} = W_1 = W_2 \) line segments \( \overline{W}_1S_1 \) and \( \overline{W}_2S_2 \) that have the same sizes and directions as \( W_1S_1 \) and \( W_2S_2 \). Consider the points \( \overline{S}_1 \) and \( \overline{S}_2 \) of space \( \overline{\Phi} \) as corresponding to \( S_1 \) and \( S_2 \) of \( \Phi \). Thus, the curve \( s \) is transformed to the curve \( \overline{s} \). The time of isolation of the point \( \overline{W} \) through light opening \( s \) is equal to the time of insolation of glass cover \( w \) through the light opening \( s \), which is equal to the time of insolation of a room that is disposed behind the light opening.

In general case, we may assume that the curve \( w \) bounds not a flat cell, but the cell of glass cover surface (for example for a spherical clerestory). In this case, for a pencil of planes with a parallelism plane \( \alpha \), we need to find such a pair of straight lines \( W_1S_1 \) and \( W_2S_2 \) that have the maximal crossing corner \( \beta_{\alpha} \) and either cross the curve \( w \) or touch the glass cover surface.

The realization of this algorithm is simplified if the external contour of window and the glass cover plane is parallel. In this case the image of curve \( s \) is easily constructed by AutoCAD.

**4 Summary**

The methods that we consider allow to calculate natural illumination and house insolation, and to find the area and form of windows. They may help to raise the energy effectiveness of buildings.
REFERENCES


OPTYMALIZACJA CECH GEOMETRYCZNYCH OKIEN
W ASPEKCIE PROJEKTOWANIA OCHRONY ENERGETYCZNEJ
BUDYNKÓW

Straty ciepła przez okna są czynnikiem bardzo istotnym z punktu widzenia bilansu energetycznego budynków. W artykule omówiono problem optymalizacji kształtu i wielkości okien związanego:
- z obliczaniem wartości będących funkcją oświetlenia naturalnego,
- z izolacyjnością budynków.

Celem badań prowadzonych przez autora jest udoskonalenie metod zaakceptowanych jako standardowe na Ukrainie.