Abstract: The paper is a sequel to the author’s earlier considerations relating to panoramic representations on a cylindrical surface. The theory of the panoramic projecting is developed in two directions distinguishing the following phenomena: panoramic projecting from single center and panoramic projecting from dispersed centers, where the center of projecting moves along the circle. It aims at better approximation of received images of objects to reality of optic noticing and impression an observer of the panorama should get walking around the platform. The center of the panoramic projecting is regarded as a geometrical counterpart of his eyes. The paper discusses resemblances and differences of geometrical aspects of the construction of both kinds of panoramas as well as their mappings received on the unreeled background.

Keywords: panorama image, cylindrical perspective, engineering graphics, computer aided drawing

Introduction

The idea of the panoramic presentation is carried on in contemporary times as panorama can be a good means of the graphical communication. The analysis of the classical method of the composition of the panoramic image, where the canvas requires the special structure - the frame which forms cylindrical shape of it, led to the idea of the creating a panoramic image on a flat background. That is received as a result of the unreeling linen on the flat surface. In general the observer of panorama does not stand in one place but moves around the panorama platform so the theory of the panoramic representation is developed in two directions distinguishing: panorama from a single center and panorama from dispersed centers (multicenter panorama).

1 Basic information

The panoramic cylindrical projecting from the single $S$ center is the projecting onto a $\tau$ background which is a rotary cylindrical surface. The $S$ center is the proper point of the $l$ axis (Fig.1a). However, the cylindrical projecting from dispersed centers is the projecting from the centre moving on the $\hat{s}$ circle which is a model of the panorama platform. The $\hat{s}$ circle is included in the plane which is perpendicular to the $l$ axis and its center is a proper point of the $l$ line (Fig.1b).
The use of the moving center of the projecting aims at better approximation of received images of objects to reality of the optic noticing and impression an observer of the panorama should have walking around the platform. Defining the apparatus of the panoramic projecting in this way a projection of any proper \( X \neq S \) point is a \( X^S \) point which is a common element of the \( \bar{\tau} \) background and the \( SX^\rightarrow \) half-line (Fig. 1a). The image of any point \( Y_{\infty} \notin l \) is the sum of two points \( Y_{\infty}^{S1} \) and \( Y_{\infty}^{S2} \) for which the straight line containing point \( Y_{\infty} \) intersects the background \( \bar{\tau} \). However, in case of the multicenter panorama a changeable \( S_X \) center of the panoramic projecting is attributed to the given proper \( X \) point cutting the \( \bar{s} \) circle by a half-plane. That half-plane has the \( l \) edge and passes the \( X \) point (Fig. 1b). Nevertheless in case of the point at infinity two centers are received.

2 The mapping of the straight line

In all graphical representations the most essential matter is the mapping of the straight line and so that problem is taken up in the panoramic mapping.

The panorama image of the straight line depends on the kind of panorama as well as the way the location of it towards the apparatus. The image of the \( m \) straight line not particularly situated towards any elements of the projective apparatus is a \( m^S \) curve of the ellipse contained in the \( \bar{\tau} \) background, in case of the single center panorama. However in case of the multicenter panorama the image of the straight line is a common part of the background and a conusoid so it is the higher degree line.

The knowledge of the mapping of points and straight lines in the panorama projecting gives theoretical possibilities of creating panoramas of different figures on the not-deformed cylindrical background. However, for the graphical mapping of effects of the projecting on the flat surface it is necessary to transform images contained in the \( \bar{\tau} \) background to their counterparts in the unreeled background. Such aim can be achieve by projecting cylindrical background from the \( S \) center onto the \( \pi \) base plane. It enables establishing projective relations between points on generatrices in this degenerate flat surface (received in that projecting) and their counterparts in the unreeled background. It was presented in case of the single center panorama [1]. Similar relations can be found in case of the multicentre panorama (Fig2).

First of all the apparatus of the representation is determined within an accuracy of isometry. So the height of the horizon \( h \) circle towards the \( \pi \) base plane is established. The size of the radius of the \( \bar{p} \) base circle and the \( \bar{s} \) circle is defined too. Next the so-called \( t_g \)
base generatrix and \( t_A \) generatrix which includes the projection of the given \( A \) point is distinguished in the cylindrical background (Fig. 2).

Fig.2: The projecting of the \( t_A \) straight line and the \( S^R t_A (P_A, H_A, W_\infty, A^S, ... \) series of points in the aim of the realization of the transformation.

Conversely the fragment of the unreeled \( \tilde{\tau}^R \) background with distinguished generatrices: the \( t^R \) base one and the \( t^R \) one as well the \( p^R \) base line and the \( h^R \) horizon line are drawn on the \( \pi \) base plane (Fig. 3). The \( \tilde{\tau}^R \) image of the unreeled \( \tilde{\tau} \) background is placed itself towards the \( p \) base circle this way that the straight line containing the \( p \) base line goes through the centre of the \( p \) base circle.

Next the \( \tilde{\tau} \) background and particularly the \( t \) and the \( t_A \) generatrices with the established \( t_A (P_A, H_A, W_\infty, A^S, ...) \) series of points, where: \( P_A \in \tilde{\tau}, H_A \in \tilde{\tau}, W_\infty \) is a top of the surface, are projected from the \( S \) center onto the \( \pi \) base plane (Fig. 3). As a result of that projecting the \( S^S t_A (S^S P_A, S^S H_A, S^S W, S^S A^S, ...) \) series of points is obtained in the \( S^S t_A \) generatrix. Then \( S^S t_A \) straight line with \( S^S t_A (S^S P_A, S^S H_A, S^S W, S^S A^S, ...) \) series of points included in it is turned around the center of the \( \tilde{\tau}^R \) straight line. After this transformation the \( O t_A \) line with the \( O t_A (O^P_A, O^H_A, O^W, O^A^S, ...) \) series of points is achieved. In a row a \( t_A^R \) straight line matching the \( \tilde{\tau} A \) one is located in the unreeled \( \tilde{\tau}^R \) background as well as \( P_A^R, H_A^R, W_\infty^R \) points included in it matching \( P_A, H_A, W_\infty \) ones. Next the \( t_A^R \) straight line with distinguished \( t_A^R (P_A^R, H_A^R, W_\infty^R, ...) \) series of points is submitted to the translation by the \( P_A^R O P_A \) vector. That geometrical action points out that the \( O t_A (O^P_A, O^H_A, O^W, O^A^S, ...) \) series of points obtained after the rotation and the \( T t \) \( T P_A = O^P_A, T^A H_A, T^W, ... \) series of points obtained as a result of the translation are projective. As they have also united homologous \( t P_A = O P_A \) points they are perspective ones. The \( Q \) point is a centre of that perspectivity.
Fig. 3: Graphical connections between the $A^{SR}$ mapping of the $A$ point on the unreeled $\tau$ background and its $S^{A}$ mapping in the projecting from the $S$ center to the $\pi$ plane.

It gives graphical connections of the image of the point received in the additional projecting from the $S$ centre onto the $\pi$ base plane with its counterpart received in the unreeled $\tau$ background.

3 Resemblances and differences in images

The presented method of the handwritten construction of the panorama image is rather laborious so panoramas of complex objects are realized by computer aiding. To achieve that the straight line is written in analytical way and next transformed to its counterpart in the unreeled background, using projective relations which were introduced earlier [2]. It enables quick drawing of the panorama image of any spatial figure shaped by using lines. It also allows the analysis of differences and resemblances in the mapping of both kinds of panoramas. The differences are visible (Fig.4,5).

![Fig.4: Comparison between the images of the cube shaped block received in single center and multicenter panoramas.](image-url)
They depend on the size and shape of the object as well as its location towards the panorama background. However, the mapping of the straight line not particularly situated towards the apparatus is a wave of sine in both kinds of panoramas. In case of horizontal straight lines ends of the sine wave belong to the $\tilde{h}^R$ horizon line on the unreeled background. The size of the amplitude of the wave depends on the height the line is situated towards the $\pi$ base plane in comparison with the height of the horizon $\tilde{h}$ circle. The closer the straight line is situated towards the level of the horizon circle the less the amplitude of the wave is.

It also depends on the distance of the line from the background. One can see that the farther the straight line is from the background the less the amplitude of the sine waves is. Besides for straight lines considerably distant from the background the difference of their amplitudes is very little so the mapping of them is similar. The same relation is found in case of the multicenter panorama.

**Conclusions**

- Both kinds of panoramas are possible to present on unreeled background.
- In case of the multicenter panorama, received drawings are closer to effects of the optic noticing so can be used as a draft of the panorama image.
- There are visible differences in graphical images of both kinds of panoramas.
In case of objects situated quite far from the background both kinds of panoramas can be used interchangeably as well as for advertisement use.

References:

PANORAMA WALCOWA - DWA PODEJŚCIA

Praca nawiązuje do wcześniejszych rozważań autora dotyczących odwzorowania panoramicznego na cylindrycznej powierzchni. Teorię panoramicznego rzutowania rozwija się w dwóch kierunkach wyodrębniając panoramiczne rzutowanie z pojedynczego środka oraz panoramiczne rzutowanie z rozproszonych środków, w którym środek rzutowania porusza się po okręgu. Ma to na celu lepszą aproksymację uzyskiwanych obrazów obiektów do rzeczywistego optycznego postrzegania i wrażenia jakie obserwator panoramy uzyskuje spacerując wokół platformy widokowej. Środek panoramicznego rzutowania uważa się za geometryczny odpowiednik jego oczu. Artykuł omawia podobieństwa i różnice geometrycznych aspektów konstrukcji obydwu typów panoram jak również ich zapisów uzyskanych na rozwiniętych tłach.