POINTS MAPPING IN THE CENTRAL-REFLEXIVE PROJECTION ON CONICAL SURFACE

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Abstract: The paper suggests the solution to the problem of seeking the mirror reflection of any point from the flank surface of a cone in relation to the eye point, which is the centre of projection, on the grounds of descriptive geometry. To accomplish this goal a couple of spatial curves is used, whose point of interception is the sought point.

Keywords: cone, mirror reflection, curved mirrors, spatial curves.

1 The goal of the dissertation

Fig.1: Cones, Bert Flugelman, Canberra, National Gallery of Australia.

It is hard to overestimate the meaning of the phenomenon of reflection, due to its common occurrence for our perception of reality, not only in the visual sense, but also in all the situations when we have to deal with the reflection of waves, e.g. sound waves, heat waves or electromagnetic waves. Due to his profession, the author is mostly interested in the

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visual trace. The problem of mirror reflections from curved surfaces is not new, the works of Glaeser [3], Szymański [4,5], Zdziarski are famous [6] as the ones which deal with the reflections from cilindrical and spherical surfaces as well as possible anamorphical transformations on these surfaces. At the same time, works dealing with mirror reflections from the surface of a cone based on the grounds of descriptive geometry, are not known to the author. The goal of this paper is to build a geometrical model, which could allow to realise the projection of the reflection of any point P which could be found in the projective space S^3 in the mirror formed on the side surface of the cone γ_2, which comprises the projection surface (Fig. 2). Such point R is to be found on the surface γ_2, that the radius, which emerges from point *P* after the reflection in R could go through the centre of the projection E (eye).

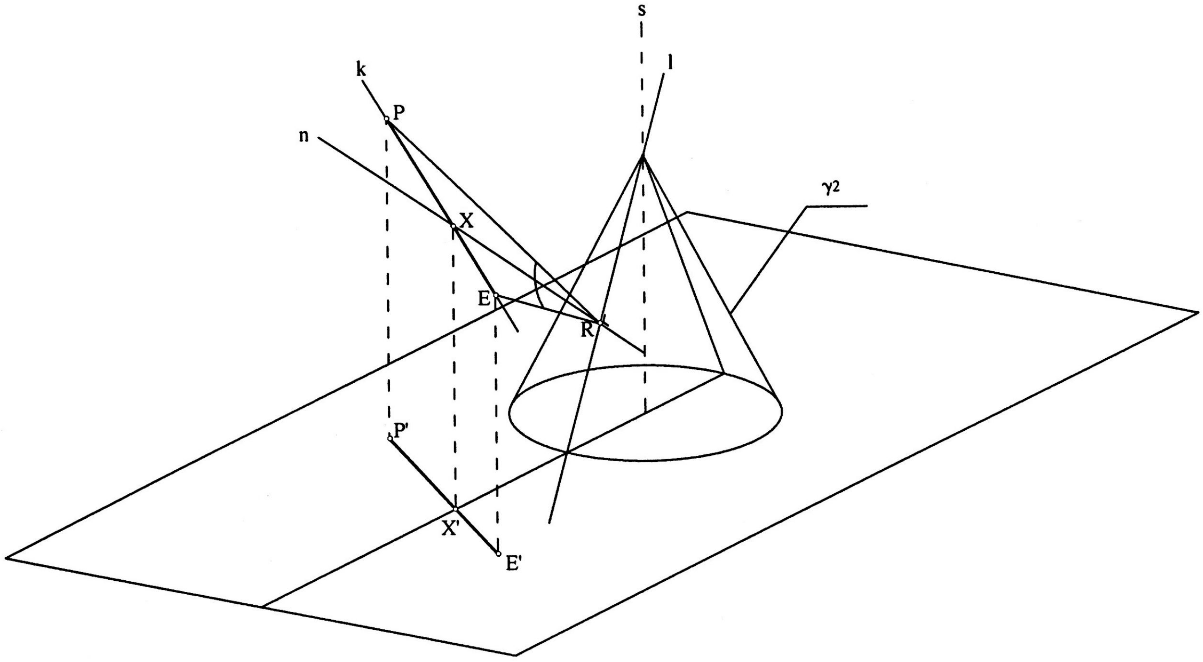

Fig.2: Denoting the elements occurring in the construction

2 Dependence between the elements of the construction

Finding a reflection point by approximate method appears to be possible on the grounds of descriptive geometry after analysing interdependences existing between elements of this geometrical situation.

Here they are:

1. R belongs to surface $\gamma_{2,}$
2. straight line perpendicular to γ_2 in point R (normal straight line n) is perpendicular to straight line l, which forms the cone by the rotation on axis s,
3. straight line n has got a common point with straight line s,
4. straight line n has got a common point with straight line k formed from points E and P (let point X be the crossing of these lines); this point lies between points P and E,
5. angles XRP and XRE are equal and are found on the same plane.

3 Geometrical construction

Finding the sought point consists of three stages. In the first stage we look for the set of points which complies with the conditions mentioned earlier in 1, 2, 3 and 4. In this way we get a set of points which can be presented as a spatial curve in the figure. In the second stage of the construction we look for the set of points which complies with the conditions mentioned in 2, 3, 4 and 5. As a result, we receive the second spatial curve. In the third stage we find point (or points) of reflection of R as the point common for both these curves.

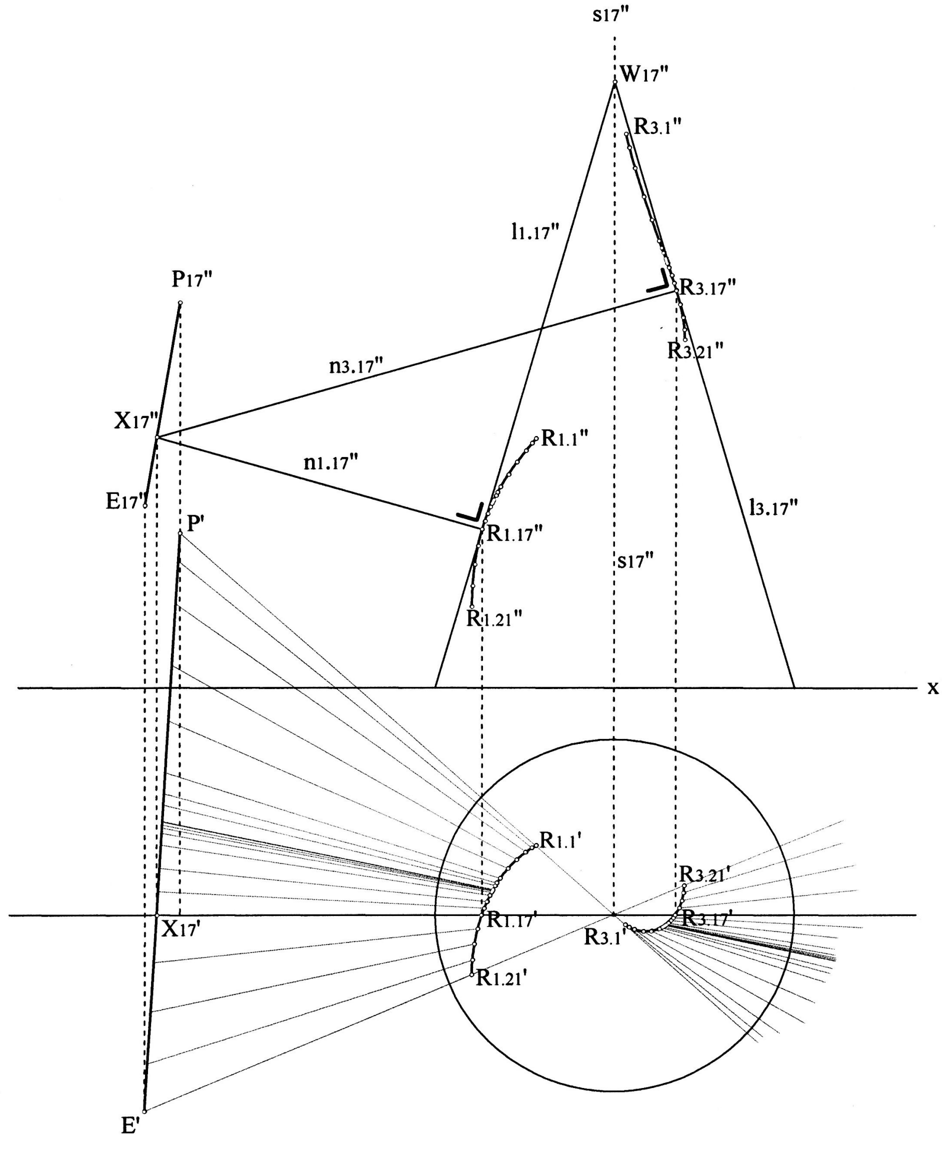

Fig.3: Construction of curves complying with the conditions 1, 2, 3 and 4

Construction 1

We have the given cone created by the rotation of slant height *l* around the straight line s with an apex in the point W, we draw it in the Monge projection firstly on the projective plane perpendicular to s in the form of a circle which is the projection of the cone cut by the projection surface. On this projective plane we also mark the projections of the given points P and E. Then we perform a series of parallel projection**s to s**. Each such projection is based on the plane formed by s and one of following points situated between P and E, that is X_1, X_2,..., X_n. On the attached example (Fig. 3) 21 such projections have been made, one of which based on point X_{17} and *s*, is presented. On each of the projections, firstly the given elements are drawn, i.e. the projections of the points P, E**,** X_n, the straight line s and of those from the pencil of straight lines l_n, which in this case become the outline of the cone. Secondly, we draw normal lines n_n to these slant heights which are contour lines of the cone going through X_n and we find the potential points R_n (in our example to the slant heights $l_{1.17}$ and $l_{3.17}$ we draw normal lines $n_{1.17}$ and $n_{3.17}$ through the point X_{17} and we receive points $R_{1.17}$ and $R_{3.17}$). The sum of the acquired potential points R_n from all the parallel projections towards *s* gives us the possibility of summing up the results to the form of a curve.

Construction 2

In the second construction, we use the data, that were already used to draw the first spatial curve: perpendicular projection to s with the circle describing the cone and the projections of the points P and E as well as the set of projections parallel to s, each of which had its own image of the points P, E, and X_n from which the normal line n_n to the slant heights of the cone l_n were traced. In order to maintain the equality of angles between the angle XRP and the angle XRE (condition 5) we will trace on each projection parallel to s the following construction of seeking the potential points R_n (Fig. 4): from the projection of the point *E* we draw a straight line perpendicular to n_n (we will name it t_n), next on it we find the distance E from n_n, the same distance is transferred to the other side of n_n along t_n and we acquire the auxiliary point E_n, which, after connecting with the point P, with a straight line gives us on the point of intersection of this straight line with the straight line n_n the sought potential point R_n. In our example, E" transferred parallely to $n_{3.17}$" gave us the point $E_{4.17}$" in the distance equal to the distance between E" and $n_{3.17}$", and after connecting P" to $E_{4.17}$" with a straight line we acquire, on the intersection point with $n_{3.17}$", point $R_{4.17}$", acting in analogical way we acquired on the straight line $n_{1.17}$" point $R_{2.17}$". This construction is sufficient to acknowledge the fact that the acquired angles X_nR_nP and X_nR_nE are equal, as they are coplanar and each of them always has a side parallel to the projective plane (X_nR_n). We observe that in this construction the acquired potential points Rn are the points of reflection from the surface of a cone, but another one, formed by the rotation around the straight line s parallel to the straight line l (in our example point $R_{4.17}$ is the real point of reflection of the point P from the surface of the cone formed by the rotation of $l_{4.17}$ around the straight line s, and the point $R_{2.17}$ is the real point of reflection of the point P from the surface of the cone formed by the rotation of $l_{2.17}$ around the straight line s). The sum of acquired potential points R_n from all the projections parallel to s gives us the second spatial curve.

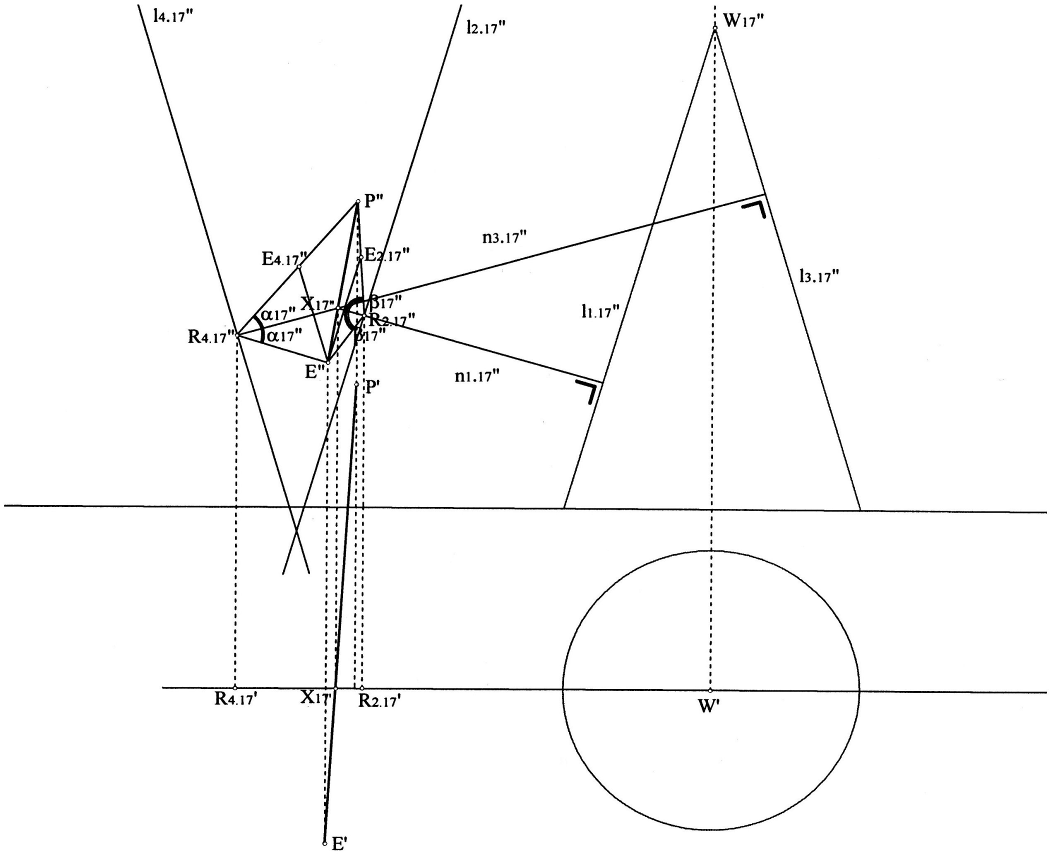

Fig.4: Construction of finding points R_n of the second curve

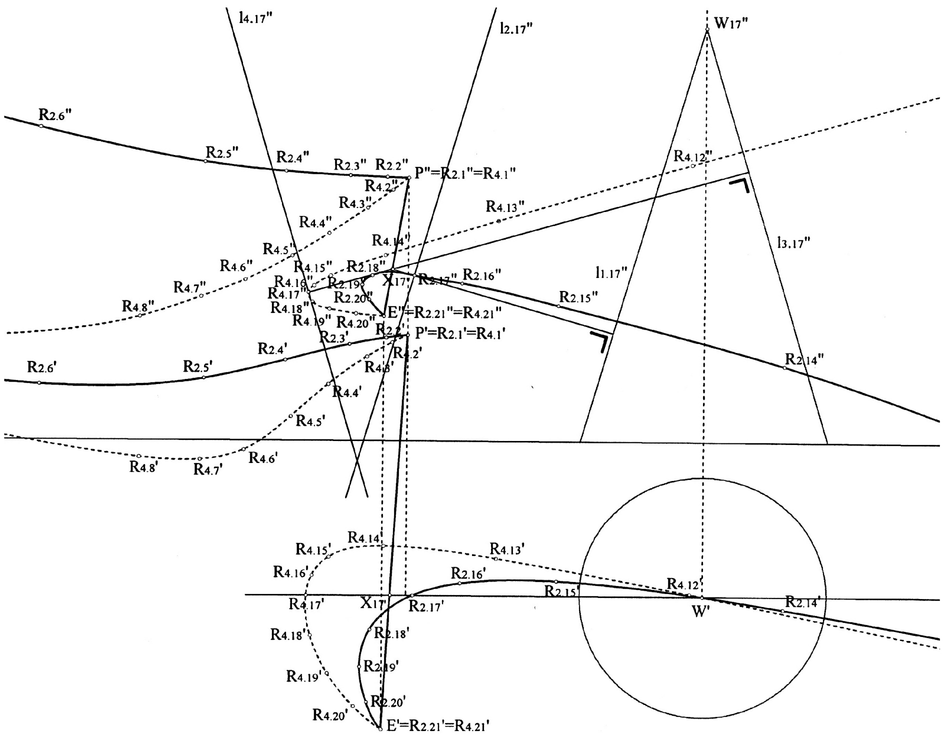

Fig.5: The image of the spatial curves complying with the conditions 2, 3, 4 and 5

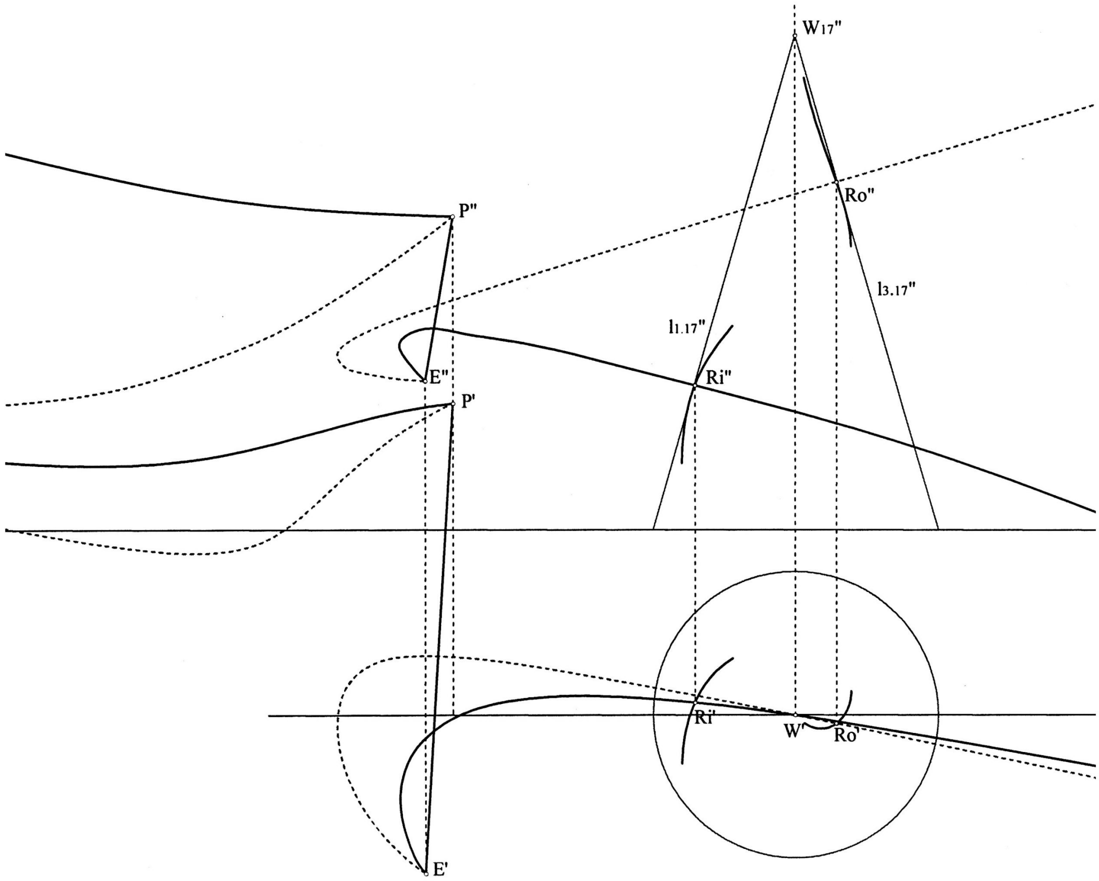

Fig. 6: The image of overlapping of both spatial curves

Construction 3

We find the sought points of reflection (Fig. 5), one on the convex surface of the cone Ri and the other one on the concave surface Ro in the points of interception of formerly acquired spatial curves.

Conclusions and further elaboration

The following paper embraces the problem of the reflection of a point from the side surface of a cone in the general sense, it does not deal with special cases of this situation, e.g. when points P and E are situated on the straight line s. Interesting seems possible after the examinations of applications of a reflecting conic surface in anamorphic shapes, especially on planes and in space.

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ODWZOROWANIE PUNKTU W RZUCIE ŚRODKOWO-REFLEKSYJNYM NA POWIERZCHNI BOCZNEJ STOŻKA

Celem tej pracy było zbudowanie modelu geometrycznego, który pozwoliłby zrealizować odwzorowanie odbicia dowolnego punktu P znajdującego się w przestrzeni rzutowej S^3 w zwierciadle utworzonym na powierzchni bocznej stożka γ_2. Należy znaleźć taki punkt R na powierzchni γ_2, aby promień wychodzący z punktu P po odbiciu w R przebiegł przez środek rzutowania E (przez oko). Odnalezienie punktu odbicia w sposób przybliżony (z dowolną dokładnością) okazuje się być możliwe na gruncie geometrii wykreślnej po przeanalizowaniu współzależności jakie występują pomiędzy elementami tej sytuacji geometrycznej. Są to:

1. R należy do powierzchni γ_2,
2. prosta prostopadła do γ_2 w punkcie R (prosta normalna n) jest prostopadła do prostej l, prostej tworzącej stożek przez obrót wokół prostej s,
3. prosta n ma punkt wspólny z prostą s,
4. prosta n ma punkt wspólny z prostą k, prostą utworzoną z punktów E i P (niech przecięciem tych prostych będzie punkt X), punkt ten leży pomiędzy punktami P i E,
5. kąt XRP i kąt XRE są równe co do wartości i współpłaszczyznowe.

Droga do odnalezienia poszukiwanego punktu składa się z trzech etapów. W pierwszym etapie poszukiwany jest zbiór punktów, który spełnia spośród wymienionych uprzednio warunki 1,2,3 i 4. Powstaje w ten sposób zbiór punktów, który możemy zapisać jako krzywą przestrzenną na rysunku. W drugim etapie konstrukcji poszukujmy zbioru punktów, które spełniałyby warunki 2,3,4 i 5. Dzięki temu otrzymujemy drugą krzywą przestrzenną. W trzecim etapie odnajdujemy punkt R jako część wspólną obu tych krzywych.