CREATING CELL VAULTS

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Abstract. The cell vaulting, which has been present in architecture since the half of the 15th century, has seen a renaissance lately. It owes its popularity to the great variety of roof structures it offers, with respect to both, its architecture and construction. The most important factors in creating cell vaults are the number of divisions and the height of joint level of a given structure.
In this paper we would like to present the geometrical methods of creating cell vaults.

Key words: cell vault, vault elevation, structure joints, joint level, cell structure

The cell vaulting came into use in northern Europe in the late Gothic period, i.e. at the turn of the 15th and 16th centuries. The cells in this type of vaulting were formed at different angles and from geometrical point of view, they were fragments of planes and not surfaces, as in other types of vaults. The light reflecting in the cells produced very interesting visual and decorative effects.

Fig1. Albrechtsburg castle in Meissen – the light reflecting off the cell vault

Modern times call for constructions of huge cubature. It makes employing large spans between supports necessary, which results in noticeable thickness of the ceiling. The most popular types of roof structures are among others: trusses (production bays, warehouses), shells (stadiums), saw-tooth roofs (factories), grillages, ribbed or beam constructions (e.g. the ceiling of the Sanctuary in Łagiewniki). Generally, the roof structures can be divided into: surface, shell and saw-tooth structures. In this paper, we would like to concentrate on another type of construction, whose working principle resembles this of the saw tooth structure. Numerous already existing buildings which employ the cell-vaulting are characterized by very interesting constructional as well as practical and aesthetic solutions.

Fig 2. The Sanctuary of God's Charity in Łagiewniki – the vault sketch; designed by prof. W. Cęckiewicz

The cell-vaulting is very rigid and durable, which puts it before other types of constructions with similar spans and parameters.

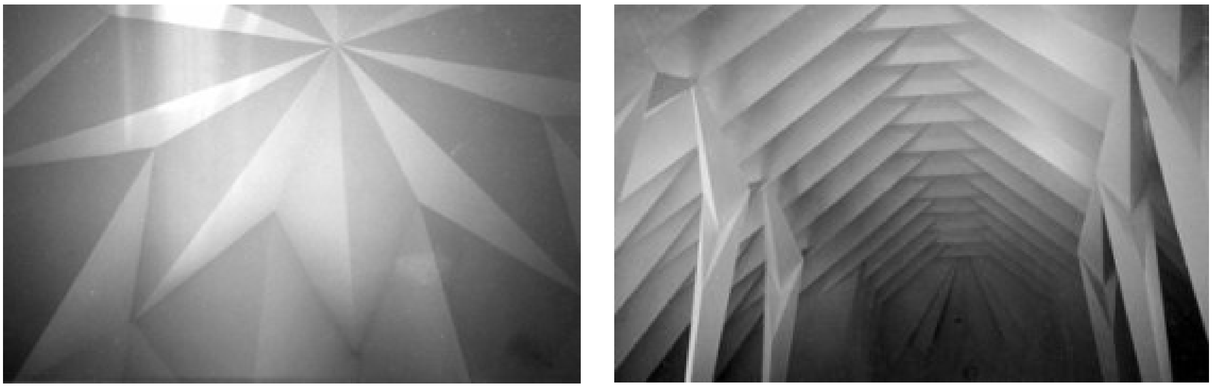

Fig 3. The cell vaults in God's Charity church in Tylmanowa (A. Salmonowicz-Olszewska, J. Rymian, A. Flga, P. Sulik, J. Sulik, J. Borkowski)

One of the most important terms is the vault elevation. By applying various proportions between the radius of the base and the elevation one can achieve diverse visual effects in a building. Small elevations of the cross-sections, resembling lenses, give a totally different effect than high structures similar in their cross-section to the parabola. It seems that the golden proportion (1 : 1,618; 1 : 0,618) or the silver one (1 : $\sqrt{2}$), are the best since they guaran

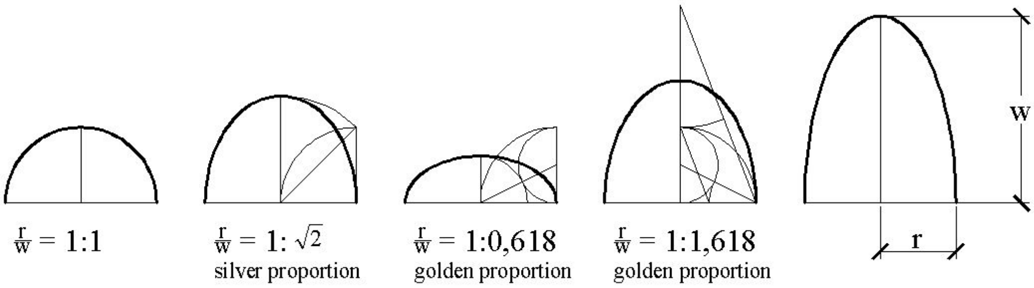

Fig 4. Different proportions of the radius and the elevation in the cell vault

tee the desirable aesthetic effect.

While creating the cell-vaulting, one should take into consideration the space which is to be covered. The use of triangular elements allows covering practically any type of space. Yet, from the aesthetic point of view, the most appropriate are the shapes of basic geometrical figures: squares, rectangles, polygons, circles or ellipses.

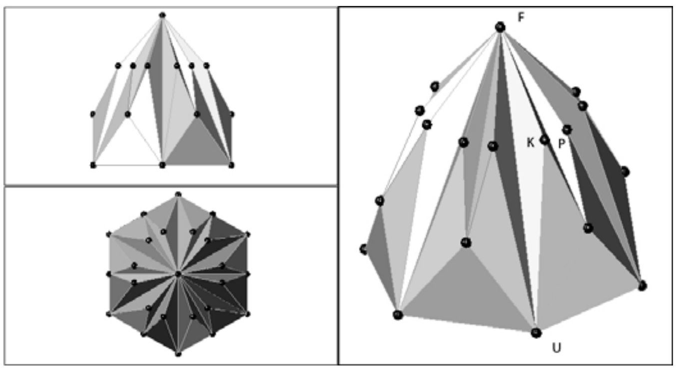

Fig 5. The cell structure with four joint levels based on a regular polygon

The first group of roof structures consists of structures with the rectangular base. The structure of a pretty simple construction, with a small number of transverse divisions and joints, based on the shape of rectangular elements is called saw-tooth structure. A similar structure, but with triangular surfaces instead of rectangular ones, is practically still the same saw-tooth structure. It becomes more complicated when one wants to increase the number of transverse cells. Such a solution has a great aesthetic value since it produces a very interesting visual effect.

A rectangle covered by a structure with a small number of joints is in fact a cuboid. Its

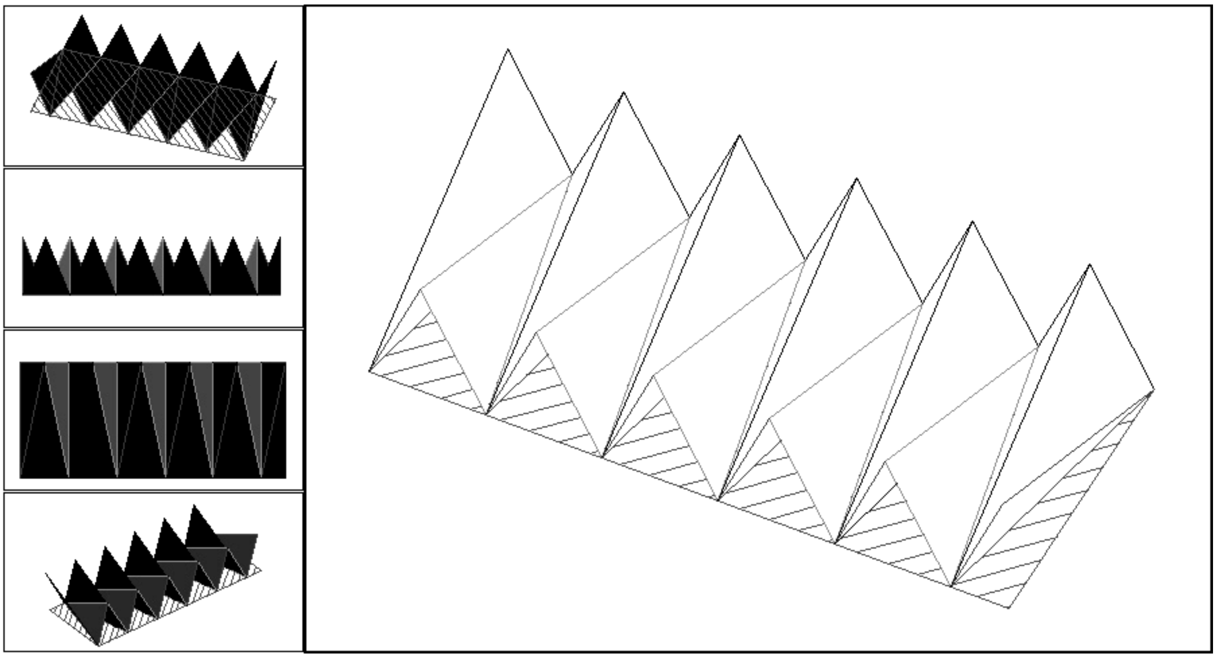

Fig 6. Forming of the vault based on the rectangle

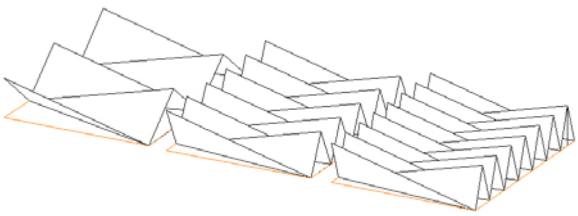

Fig 7. Cell structures based on the rectangular plan with different number of elements

distinctiveness depends on the proportion between the span and height of its elements. If the ratio is small, the roof structure resembles a corrugated plane. If the ratio is big, the depth of the structure is also big, which together with sharp edges of the elements produces a highly dramatic effect.

The more divisions of the transverse axis into cells and the more distinctive they are (owing to the height of particular elements), their cross-section more and more resembles the cylinder (a tube with the wall thickness equal to the height of particular cells). This is true in case of cells and divisions with two planes of symmetry. If there is one plane of symmetry placed along the cell span axis – the vertical axis of the cross-section, the structure will have the shape of a conical curve: a parabola, a hyperbola, an ellipse, or simply an arc. The shape of the roof is then dependent on the length of particular cells.

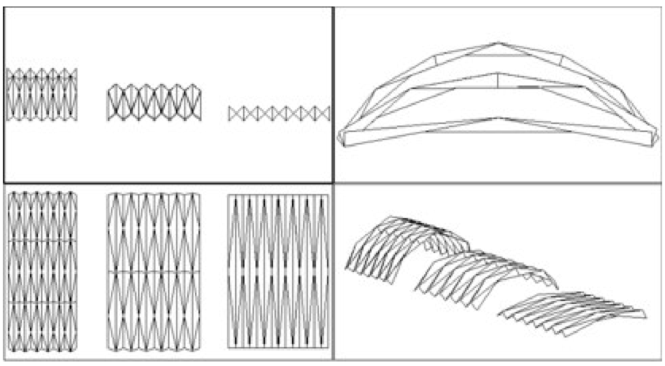

Fig 8. Making the roof structure more cylinder-like by increasing the number of cells in the vault's cross-section

Constructing of the structure without the cross-section symmetry axis results in much more complicated shapes. The rule of creating the structure in this case is also based on alternating of the length and slope of particular vault edges.

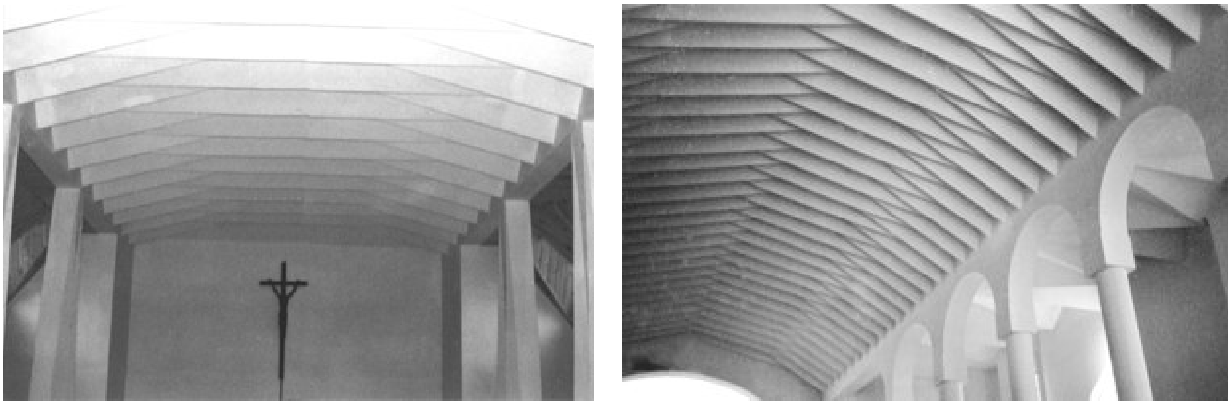

Fig 9. Different degrees of cylinder-like quality of the cell vaults as exemplified by the vaults in Jesus' Holy Heart church in Pliszczyn (E. Ryś, A. Flaga, A. Sidor) and Blessed Karolina Kózkówna church in Tarnów-Rzędzinie

Since the construction of the cell vaulting based on the square plan is very similar, with respect to the method and possibilities, to the one based on the rectangle (and – if the symmetry of its sides is the starting point – to the regular polygon), it will not be analyzed separately in this paper.

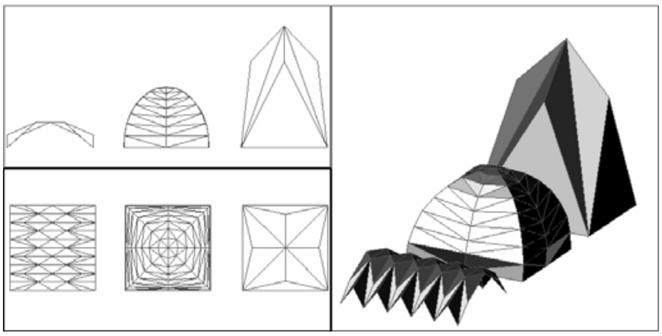

Fig 10. Examples of cell vaults based on the square

Another type of the cell vaulting, which gives the architect great possibilities to use his imagination, is the roof structure based on regular polygons.

One should start the construction with determining the number of sides of the polygon (i.e. the shape of the room or building to be covered by the vault). Another step is to calculate the angle of the figure to be "copied". Then, the number of joint levels should be established, regarding the span of the circle circumscribed on the polygon as well as the height of the roof structure. These elements will affect the level of complexity and the visual effect of the structure. Its distinctiveness will depend on the difference of height between the joint levels. Using regular polygons, one can design not only spherical vaulting, but also convex and concave ones.

There is one more group of cell vaults, namely the one based on irregular plan or with irregular shape of cells (no division between elements – axes, copied elements). Because of high complexity and diversity of such structures, they will not be discussed further in this paper.

With respect to their geometry, the structures in question can be classified in many different ways. Yet, for the sake of this analysis we need one more classification, namely into structures of I and II degree. The II degree structures can be created of one sheet of paper, without cutting it into pieces or using glue.

Constructing the cell vault, one should remember the following:

- while creating the structure based on the rectangle, the number and height of the element divisions should be determined;
- the formula $\alpha = 2\pi/m$, where m – is the number of polygon sides, α – is the angle of the isosceles triangle "copied" in the polygon – is the basis for creating the vaults constructed on regular polygons;

- determining the vault elevation, the height and difference between joint levels, is important with respect to the structure's visual effect;
- the number of divisions and structure axes is important with respect to its distinctiveness.

Having prepared this preliminary plan, one can start creating the structure. There are two methods of constructing the vault based on the regular polygon. Each of them gives a different final effect.

The first one lies in constructing two curves *AB* and *CD* meeting at the apex of the construction (*A* = *C*), which is placed above the centre of area of the regular polygon, at the height equal the elevation *h*. The curve with end *B* is fixed at the polygon's apex, and the one with end *D* – in the middle of the segment connecting two adjacent apexes of the polygon *BE*. Having divided each of the curves *AB* and *CD* into the same number of elements and connected the points accordingly, as in the picture, one receives the net of cells.

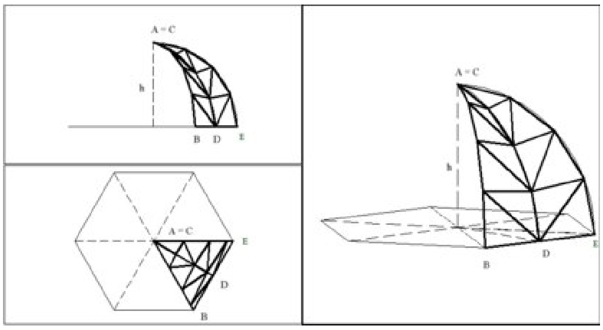

Fig11. The first method of creating of the cell structure based on the regular polygon

Another method, which results in more distinctive cell vaulting, lies in longitudinal division. In this case, there is also a polygon as the base and the elevation *h*. The difference consists in adding other joint levels – points (joints) *C* and *D* placed at such a level are at the same height. Any time, when a new joint level is added, new points are created at the same height (e.g. *G* and *H*, etc.). For the construction presented in the picture, it is the highest point – *B*.
Having determined all joints, one should connect the basic points (i.e. apexes of the polygon – *A* with the first joint level – points *C* and *D* and one of the points at another level - *B*). After connecting the points as in the picture (…*ABC* and …*ABD*), we receive a fragment of the cell vaulting in shape of two triangular planes with common bottom edge *AB*.

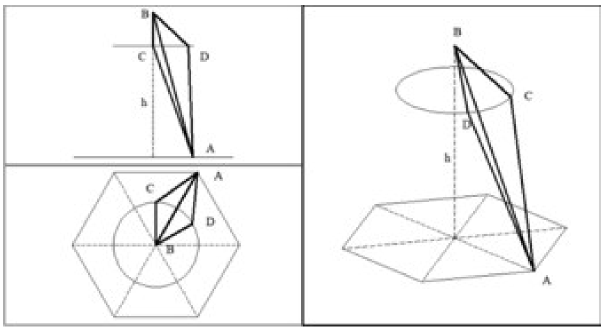

Fig 12. The second method of creating of the cell structure based on the regular polygon

The diversity of the structures is practically unlimited. Taking into consideration alternating lengths of cell edges, one can distinguish another group of structures – irregular ones. There are a lot of vault types, which can not be employed inside a building as ceilings. On the other hand, numerous structures, which are theoretically possible and can be computer generated,

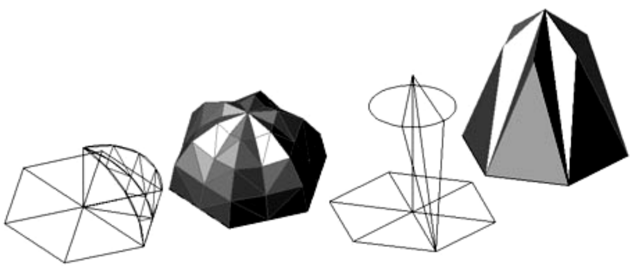

Fig13. Examples of cell vaults constructed with the use of the first and second method

will not be put into practical use because of aesthetic reasons.
Yet, the rational approach to the problem will contribute to creating the most aesthetic and functional structure, which could ever be constructed.

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