# **DIVISION OF COUPLER PLANE**

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**Abstract.** In this work a division of coupler plane of the crank-rocker four-bar linkage has been presented. This division is made by means of four curves: moving centrode, constant centrode, moving curve of rotation centres and constant curve of rotation centres. The method of determining these curves has been discussed. Assuming the coupler points in some region of the coupler plane can determine the coupler curves with characteristic points. This article presents some examples of these curves.

Key Words: coupler plane, coupler curve with characteristic points, centrode.

### 1. Introduction

The search of curves about given shapes on the coupler plane has been carried by researchers from all over word for many years [1]. It is related to the fact, that individual links of the mechanisms must do exactly given motion in precision planned parts of space. That given mechanism does charge it with a task the trajectories of moving its links must be in advance to research. This work is concerned with the division of coupler plane of the crankrocker four-bar linkage.

### 2. Division of coupler plane

Fig.1shows the crank-rocker four-bar linkage with the following terms assumed:

- driven link 1 (input) crank; performs full revolution
- link 3 (output) rocker; performs oscillation motion
- link 0 base
- link 2 coupler



Figure 1: The crank-rocker four-bar linkage

If the given point F is rigid join with the coupler 2 (Fig.1), than during full rotation of crank the moving path of the point F is so called *coupler curve*. It depends on the position of the point F on coupler plane the coupler curves can have different shape.

For given four-bar mechanism the position of points on the coupler plane can be precisely determined which trajectories of movements being curves with characteristic points. In this way the following curves can be determined

- moving centrode  $c_r$ , and constant centrode  $c_s$ ,
- moving curve of rotation centres  $k_r^{\beta}$  and constant curve of rotation centres  $k_s^{\beta}$ .

The *constant centrode*  $c_s$  create instantaneous rotation centres of links 1 and 3, so the intersection points of straight line form their links (Figure 2). The *moving centrode* is determined to make inversion of mechanism. To this end the link 2 is assumed as a basis link and the position of link 0 is changed. The intersection points of straight lines which include links 1 and 3 are determined for full rotation of the crank 1 (Figure 3).



Figure 2: The constant centrode



Figure 3: The moving centrode

For the obtained results the constant and moving curves of rotations centre take the following:

- 1/ The position of optional links of the mechanism was assumed. An optional point  $K_0$  which lies on the constant centrode was adopted. (Fig.4) /curves c's, c's its two branches of the constant centrode  $K_0 \in c's/$
- 2/ The line determined by points A and K<sub>0</sub> was recorded as s
- 3/ The points B<sub>i</sub>, B<sub>j</sub> are determined on the trajectory of the kinematic pair B, which are symmetrically arranged in relation to line s.
- 4/ For points B<sub>i</sub>, B<sub>i</sub> the position of the point C which was recorded as C<sub>i</sub>, C<sub>i</sub>, is determined
- 5/ For point D and point  $C_s$  (middle point of the interval  $C_iC_i$ ) the line t was drawn.
- 6/ Intersection point of the lines s and t was recorded  $K_{\beta}$ . This point belongs to constant curve of rotations centre. Point  $K_{\beta}$  is a coupler point for two positions of the coupler. This means, that coupler point (during full rotation of the crank) goes through this place twice
- 7/ The point  $K_{\beta}$  rigid join with the link 2 is moving to input position of the link 2. This point belongs to the moving curve of rotations centre and is determined by M.

Assume the next points on the constant centrode and repeat steps from 1 to 7, these are determining points of the moving curve of rotations centre.



Figure 4: The constant and moving curves of rotations centre  $k_{s}^{\beta}$ ,  $k_{r}^{\beta}$ 

The angle  $\beta$  can assume values from 0 to  $\pi$ . Limiting courses of the moving curves of rotations centre are the most interesting because all the rest of the curves lie between these curves. For  $\beta = 0$  the curve  $k^s_0 = c_s$ ,  $k^r_0 = c_r$ , this means, that the constant and moving centrode are the special case of the constant and moving curves of rotations centre. At the mentioned curves split the coupler plane into parts:

- a region included between curves k<sup>π</sup><sub>s</sub>, c<sub>s</sub> is the geometrical place of double points of coupler curves;
- a region included between curves  $k_{r}^{\pi}$ ,  $c_r$  is the geometrical place of the points, which draw couple curves with double points.

Within the frames of the introduced division one can distinguish geometrical place of the points, where the moving part are the coupler curves with characteristic point.

## 3. The coupler curves with characteristic points



Figure 5: The division of coupler plane over curves  $k_{r}^{\pi}$ ,  $\dot{c_{r}}$ ,  $\ddot{c_{r}}$ 

The assumed coupler points:

- on the moving centrode are determining curves with sharp points (Fig.6a)
- in points F<sub>3</sub> and C (the intersection points of the branches c'<sub>r</sub>, c"<sub>r</sub> of moving centrode) the curves with second sharp points are drawn (Fig.6b)
- in region between curves c'r, c"r curves with one intersection point are determined (Fig.6c)
- on the curve  $k_r^{\pi}$  between points  $F_1$  and  $F_2$  (the intersection points of the curves  $\dot{c_r}$ ,  $k_r^{\pi}$  and  $\ddot{c_r}$ ,  $k_r^{\pi}$ ) curves with point of osculation are determined (Fig.6d)
- in points  $F_1$  and  $F_2$  the needle curves are drawn (Fig.6e)
- in mark region between curves k<sup>π</sup><sub>r</sub>, c<sup>'</sup><sub>r</sub>, c<sup>'</sup><sub>r</sub>, c<sup>'</sup><sub>r</sub>, curves with two intersection points are determined (Fig.6f)
- outside mark region and outside curves  $k_{r}^{\pi}$ ,  $\dot{c_r}$ ,  $\dot{c_r}$ , curves without characteristic points are drawn



Figure 6: Coupler curves with characteristic points

### 4. Conclusions

The method of division of coupler plane which was presented is a base to research of coupler plane of the crank-rocker four-bar linkage. There researches are conducted in nondimensional way [3,4]. In order to determine coefficients which are ratio length of particular link to length of the driver link. The influence of changes of these coefficients to division of coupler plane was studied.

## References

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### PODZIAŁ PŁASZCZYZNY SPRZĘŻONEJ

W pracy przedstawiono badania płaszczyzny łącznikowej korbowo-wahaczowego czworoboku przegubowego. Podziału płaszczyzny łącznikowej dokonano wykorzystując następujące krzywe: centrodie ruchomą i stałą oraz ruchomą i stałą krzywą środków obrotu. Sposób otrzymywania tych krzywych zaprezentowano w pracy. W wyniku podziału płaszczyzny łącznikowej uzyskano obszary, w których punkty łącznikowe wykreślają krzywe łącznikowe z punktami charakterystycznymi. Przykłady tych krzywych zamieszczono w pracy.

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