POLYHEDRAL COVERS BASED ON L-SYSTEM FRACTAL CONSTRUCTION

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Abstact. The paper includes an analysis of L-system as a formalism used to describe plants growth, which in time has become a general method of creating fractal objects. A modification of L-system, enabling generation of 3-D structures, which could be applied as the supporting construction for polyhedral covers of architectural objects is proposed in this study. The paper presents a gallery of geometric forms which application in architecture creates an alternative for domes and other traditional types of roofs.

Key Words: fractal, L-system, polyhedral covers

1. Introduction

Fractal geometry is one of the youngest theories of contemporary mathematics, which developed thanks to the advance of computer technology. Although Cantor, van Koch and Sierpiński discovered the first fractal objects at the beginning of the 20th century, the real development of the theory of fractals was started in the 70s by a French mathematician of Polish origin - Benoit Mandelbrot. The use of a computer enabled him to create, among the others, a graphical image of the most famous object of contemporary mathematics, which came to be called Mandelbrot's set, after its creator.

2. L-systems

One of the basic methods of generating fractal objects is Lindenmayer system called L-system for short. Its creator, Aristid Lindenmayer, while carrying out research in biology, created in 1968 a formalism used to describe plants growth. Along with the development of the theory, the L-system has been enriched with geometric aspects and has become a universal tool not only for modelling plants, but also for creating fractals which shape is a function of time.

The basic idea of the L-system is a rule of rewriting, also called a rule of replacing. Its operation can be presented in terms of intuition by means of the following example:

Example 1.

Let's have a string built of elements a and b, which can occur many times in the string. Each element is associated with one rewriting rule P. The notation P: $a \rightarrow b$ means that element a is replaced with element b, and the notation P: $b \rightarrow ab$ means that element b is replaced with a two-element string ab. The rewriting process starts from distinguished string called the axiom ω. Assumptions constructed in this way make it possible to generate the following example string of elements. [1]

Another idea, besides the rule of rewriting, of L-systems is a method of notation the fractal's structure, based on graphical interpretation of a string of characters. This method is known in literature under the name of the turtle graphics, whose creator was Seymour Papert. The main idea of the turtle graphics is graphical interpretation of a string of characters in the form of commands given to a specially trained turtle. The method's tool is the turtle's tail which draws straight lines on the plane according to the received commands. A short

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list of commands by means of which one can create the simplest fractals is as follows:

F - move forward a step of constant length > 0
draw a line segment from the previous to the new position

f - move forward a step of constant length > 0 ,
without drawing a line

+ - turn left by constant angle δ

– - turn right by constant angle δ

[- meeting this command causes that the turtle's current state is remembered

] - meeting this command makes the turtle come back to the state before the symbol [

[...] - between these symbols occur commands defining the construction of the branches [1]

Example 2.

axiom ω : F

rewriting rule P : $F \rightarrow F [+ F] F [- F] F$

parameters $+ -$: $\delta = 30^o$ (1.1)

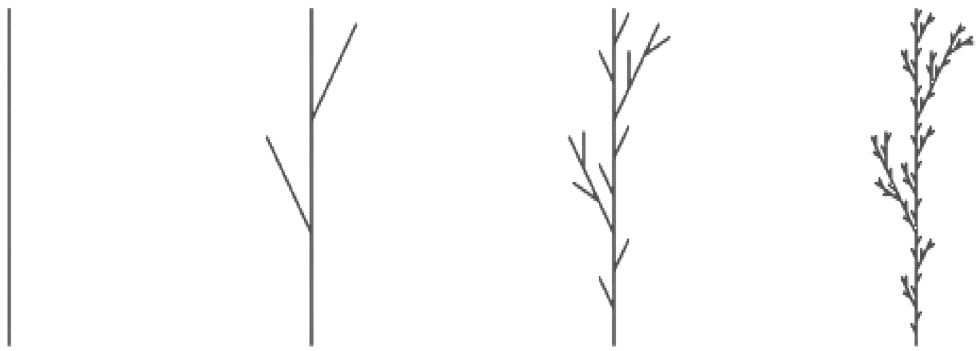


Figure 1: L-system generating a fractal resembling a weed

By means of L-system it is also possible to generate classical fractals for example van Koch curve or Giuseppe Peano curve. This is an example of a curve filling a square, which does not cross itself. [2]

Example 3.

L-system parameters for van Koch curve:

axiom ω : F

rewriting rule P : $F \rightarrow F + F - - F + F$

parameters $+ -$: $\delta = 60^o$ (1.2)

Figure 2: Van Koch's curve

Example 4.

L-system parameters for Peano curve:

axiom ω : F

rewriting rule P : $F \rightarrow F+F-F-F-F+F+F+F-F$

parameters $+ -$: $\delta = 90^o$ (1.3)

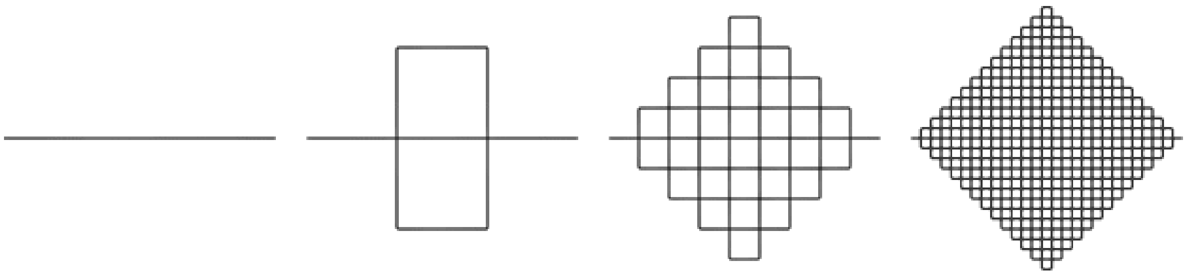


Figure 3: Peano's curve

Traditionally, however, L-systems are used to describe plants growth. A reverse activity, i.e. using L-systems to create fractals, gives examples of astonishing similarity of the generated structures to objects existing in nature.

Example 5.
L-system parameters of fractal plants [1]

Example 5a
ω: X
$X \rightarrow F[+X]F[-X]+X$
$F \rightarrow FF$
$\delta = 20^o$
$n = 7$ (1.4)

Example 5b
ω: X
$X \rightarrow F[+X][-X]FX$
$F \rightarrow FF$
$\delta = 25{,}7^o$
$n = 7$ (1.5)

Example 5c
ω: X
$X \rightarrow F-[[X]+X]+F[+FX]-X$
$F \rightarrow FF$
$\delta = 20^o$
$n = 7$ (1.6)

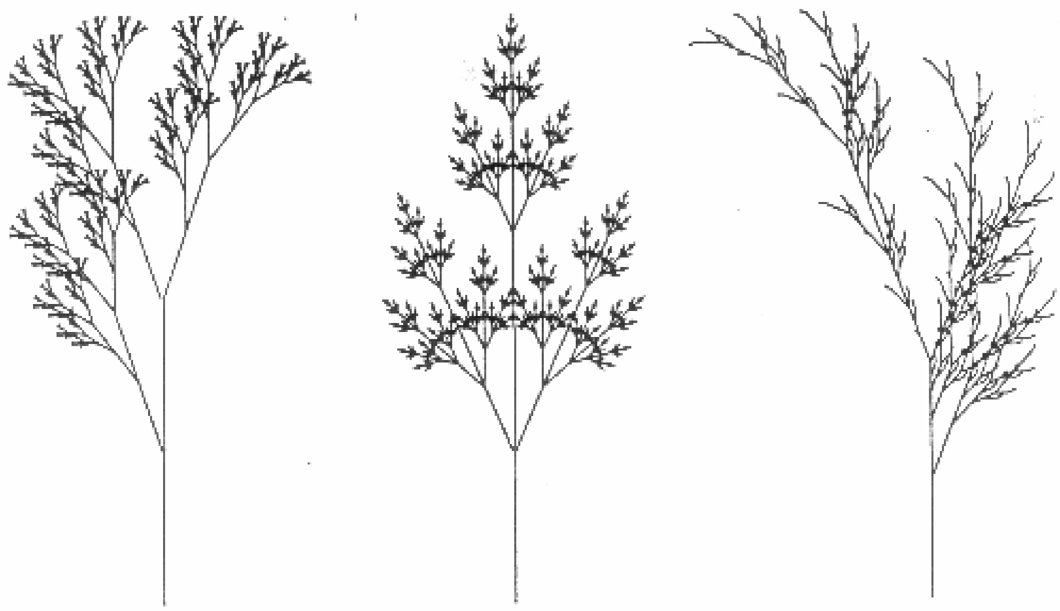


Figure 4: Examples of L-system fractal plants (a, b, c) [1]

It should be noted that in the presented examples, during the application of rewriting rules an automatic graduation of given elements takes place, which is characteristic of the simplest non-parameterized L-systems used to generate plane fractals.

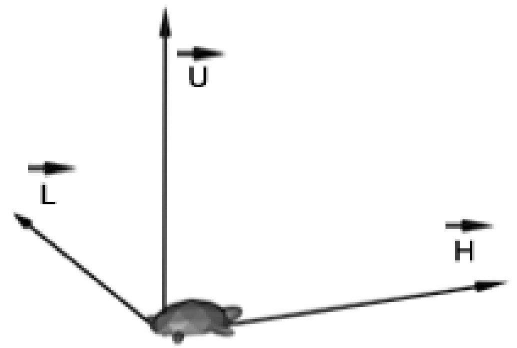
3. L-system 3-D

The interpretation of the turtle graphics can be realized in three-dimensional space. The concept of orientating the turtle in space consists in outlining three vectors H, L, U indicating: H – (heading) the turtle's head, L – (left) direction to the left, U – (up) direction up. [1]

$\vec{H}$ (heading)

$\vec{L}$ (left) 0

$\vec{U}$ (up)



The complete list of commands used in three-dimensional turtle graphics is as follows:

F(a) – move forward a step of constant length a>0, draw a line segment from the previous to the new position
f (a) – move forward a step of constant length a>0, without drawing a line
+ – turn around vector U by angle α (the turtle turns left)
− – turn around vector U by angle α (the turtle turns right)
& – turn around vector L by angle β (the turtle rises)
$\wedge$ – turn around vector L by angle β (the turtle goes down)
/ – turn around vector H by angle γ (the turtle turns right around its own axis)
\ – turn around vector H by angle γ (the turtle turns left around its own axis)
[– meeting this command causes that the turtle's current state is remembered
] – meeting this command makes the turtle come back to the state before the symbol [
[...] – between these symbols occur commands defining the construction of the branches [1]

It should be noted that the commands given to the turtle concern the orientation connected with the local co-ordinate system in which the turtle moves and which moves along with the turtle. For the needs of this study it has been assumed that initially, the turtle's head is turned upwards, thus the command F means movement upwards, and commands & or $\wedge$ mean an inclination by angle β The parameters of an example of L-system fractal plant generated by the author are given below:

Example 6.
ω: F(a)
P: F(a)$\rightarrow$F(a)[$\wedge$ F(0,5a)] [\ $\wedge$ F(0,5a)] [\ \ $\wedge$ F(0,5a)] [/ $\wedge$ F(0,5a)] [/ / $\wedge$ F(0,5a)]
$\wedge = 60^{o}$
$\backslash / = 72^{o}$
n = 3 (2.1)

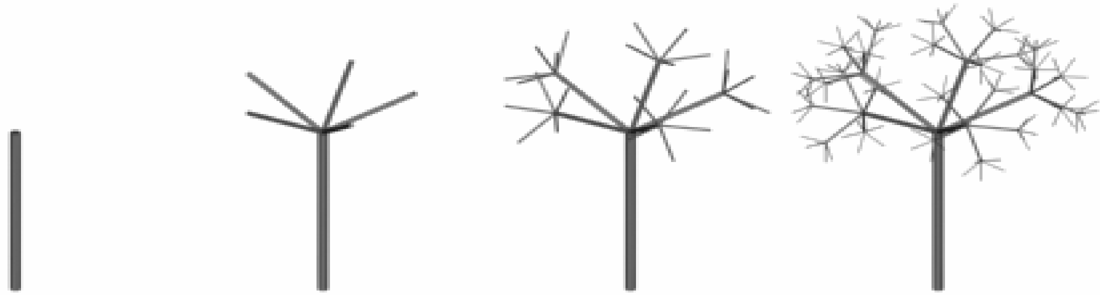


Figure 5: Successive stages of creating L-system fractal construction

The obtained L-system fractal structures show resemblance to existing inflorescences of plants from the Umbellar species in nature. The vertices of the highest 'branches' – the last stage of the rewriting process can provide the basis for unknown so far poliedric structures of fractal genesis. The structure formation process is in two stages:

– creating a network by connecting the nearest vertices
– filling the network with polygons, most frequently triangles and quadrangles

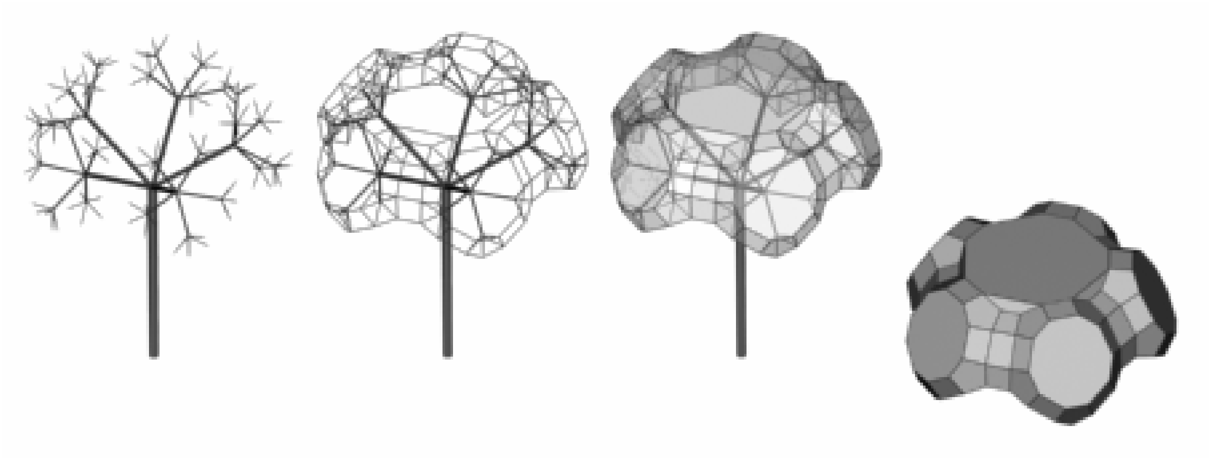


Figure 6: Successive stages of forming a poliedric structure based on L-system fractal construction

Depending on the accepted formula of the L-system, the author obtained examples of poliedric structures that can be used as roofs of architectural objects. Selected examples are presented in Table 1. results are shown in Figures 7-14. The typology and basic parameters

Table 1

Structure	Vertices	Edges	Faces					
			3-angles	4-angles	5-angles	6-angles	10-angles	Σ
Lf 3	27	42	9	3		4		16
Lf 3+	64	150	75	15				90
Lf 4	64	112		49				49
Lf 4+	125	280	112	48				160
Lf 5	125	200		45	25		51	121
Lf 5+	216	520	215	90				305
Lf 6	216	366		108		43		151
Lf 6+	343	804	252	78		48		378

The models of L-system fractal strucures and poliedric structures have been made by the author with the aid of the computer software Autodesk VIZ 4

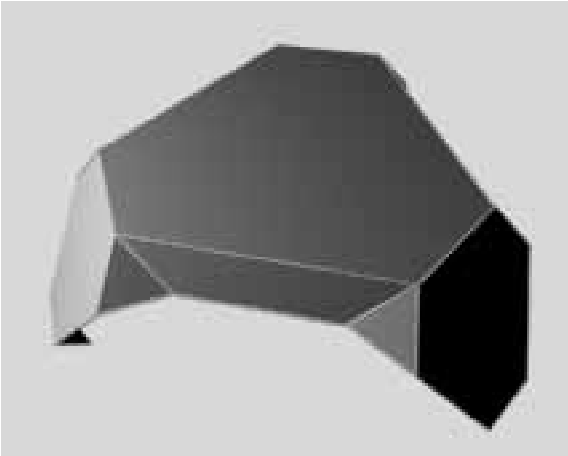


Figure 7: Lf 3

Example 7.

ω: F(a)

P: F(a) → F(a) [∧ F(0,5a)] [\ ∧ F(0,5a)] [/ ∧ F(0,5a)]

∧ = 60°
\ / = 120°
n = 3 (2.1)

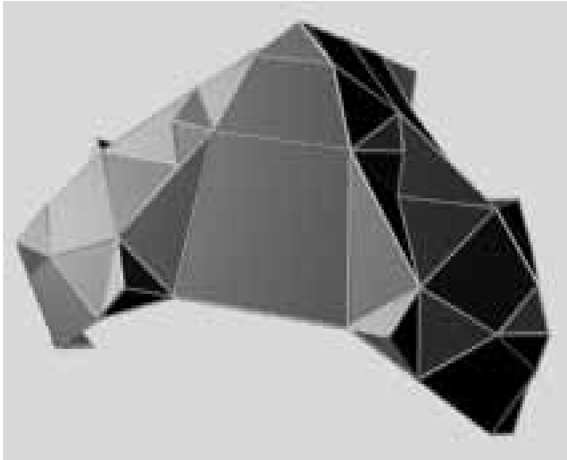


Figure 8: Lf 3+

Example 8.

ω: F(a)

P: F(a) → F(a) [F(0,5a)] [∧ F(0,5a)] [\ ∧ F(0,5a)] [/ ∧ F(0,5a)]

∧ = 60°
\ / = 120°
n = 3 (2.2)

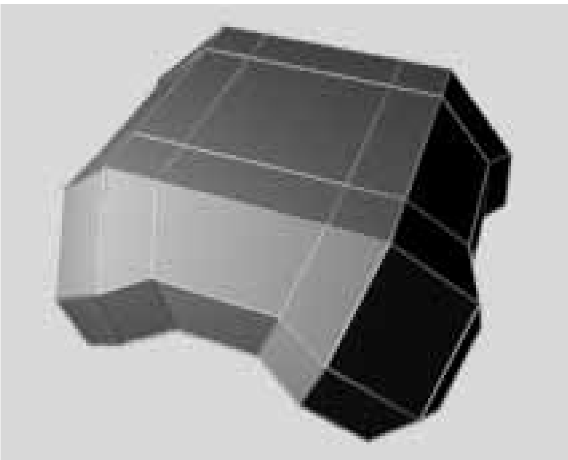


Figure 9: Lf 4

Example 9.

ω: F(a)

P: F(a) → F(a) [∧ F(0,5a)] [\ ∧ F(0,5a)] [\ \ ∧ F(0,5a)] [/ ∧ F(0,5a)]

∧ = 60°
\ / = 90°
n = 3 (2.3)

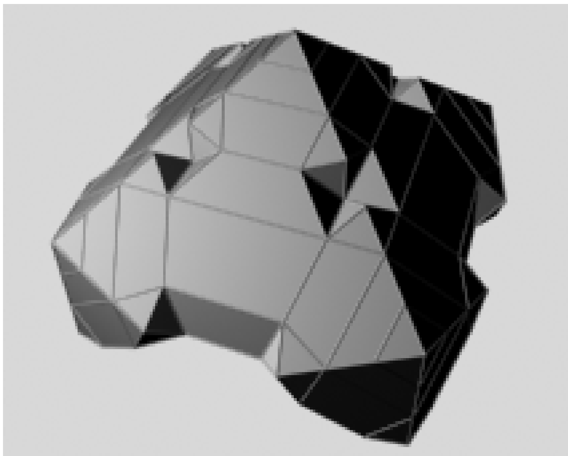


Figure 10: Lf 4+

Example 10.

ω: F(a)

P: F(a) → F(a) [F(0,5a)] [∧ F(0,5a)] [\ ∧ F(0,5a)] [\ \ ∧ F(0,5a)] [/ ∧ F(0,5a)]

∧ = 60°
\ / = 90°
n = 3 (2.4)

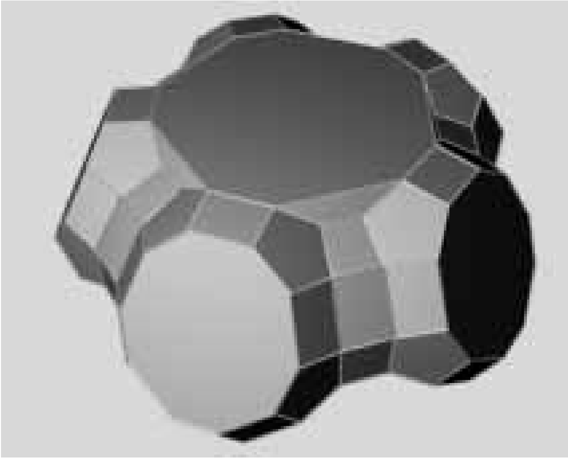


Figure 11: Lf 5

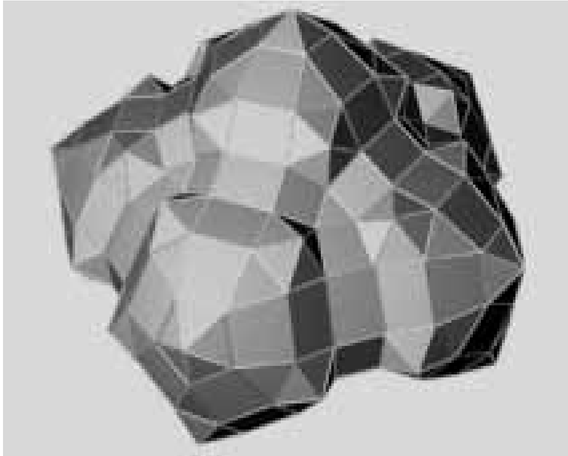


Figure 12: Lf 5+

Example 11.

ω: F(a)

P: F(a)→F(a)[∧ F(0,5a)] [\ ∧ F(0,5a)]
[\ \ ∧ F(0,5a)] [/ ∧ F(0,5a)] [/ / ∧ F(0,5a)]

$\wedge = 60^o$
$\backslash / = 72^o$
n = 3 (2.5)

Example 12.

ω: F(a)

P:F(a)→F(a)[F(0,5a)][∧F(0,5a)][\∧F(0,5a)]
[\ \ ∧ F(0,5a)] [/ ∧ F(0,5a)] [/ / ∧ F(0,5a)]

$\wedge = 60^o$
$\backslash / = 72^o$
n = 3 (2.6)

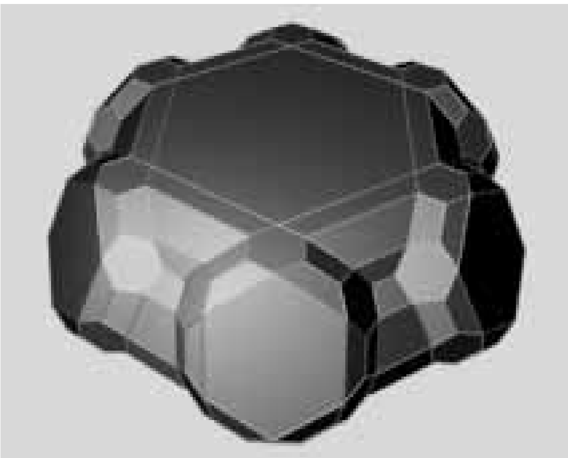


Figure 13: Lf 6

Figure 14: Lf 6+

Example 13.

ω: F(a)

P: F(a)→F(a) [∧ F(0,5a)] [\ ∧ F(0,5a)]
[\ \ ∧ F(0,5a)] [/ ∧ F(0,5a)] [/ / ∧ F(0,5a)]
[/ / / ∧ F(0,5a)]

$\wedge = 60^o$
$\backslash / = 60^o$
n = 3 (2.7)

Example 14.

ω: F(a)

P: F(a)→F(a) [F(0,5a)] [∧ F(0,5a)]
[\ ∧ F(0,5a)] [\ \ ∧ F(0,5a)] [/ ∧ F(0,5a)]
[/ / ∧ F(0,5a)] [/ / / ∧ F(0,5a)]

$\wedge = 60^o$
$\backslash / = 60^o$
n=3 (2.8)

References

[1] Prusinkiewicz P., Lindenmayer A.: *The Algorithmic Beauty of Plants*. Springer Verlag, New York 1990.

[2] Peitgen H.-O.,Jurgens H., Saupe D.: *Granice chaosu Fraktale*. Wydawnictwo Naukowe PWN, Warszawa 1995.

PRZEKRYCIA WIELOŚCIENNE OPARTE NA L-SYSTEMOWEJ KONSTRUKCJI FRAKTALNEJ

Geometria fraktalna jest jedną z najmłodszych teorii współczesnej matematyki, której rozwój dokonał się dzięki zastosowaniu technik komputerowych. Chociaż pierwsze obiekty fraktalne zostały odkryte przez Cantora, van Kocha, Sierpińskiego na początku XX wieku, właściwy rozwój teorii fraktali został zapoczątkowany w latach 70-tych przez francuskiego matematyka polskiego pochodzenia Benoita Mandelbrota. Zastosowania komputera pozwoliło między innymi na wizualizację najsłynniejszego obiektu współczesnej matematyki nazwanego od nazwiska jego twórcy zbiorem Mandelbrota.

W 1968 roku biolog Aristid Lindenmayer stworzył formalizm służący do opisu wzrostu roślin wykazujących cechy fraktalne. Formalizm zwany L-systemem w połączeniu z zastosowaniem technik komputerowych pozwolił na skonstruowanie wielu modeli roślin i monitorowaniu procesu ich wzrostu. Metoda ta polega na zastosowaniu reguły podstawiania i sposobie konstrukcji zwanej w literaturze grafiką żółwia. Przepisywanie jest techniką polegającą na zamienianiu części prostego początkowego ciągu znaków zgodnie z ustalonym zbiorem reguł przepisywania.

Wykorzystanie tej metody w przestrzeni 3-wymiarowej pozwala na tworzenie komputerowych modeli fraktalnych konstrukcji imitujących budowę roślin np. z rodziny baldaszkowatych. Dzięki wykorzystaniu technik komputerowych ułatwiona jest zmiana parametrów konstrukcji "fraktalnych roślin". Możliwe jest również generowanie wyższych niż w obiektach naturalnych stopni iteracji.

Uzyskane przykłady "fraktalnych roślin" mogą być wykorzystane jako konstrukcje wsporcze dla wielościennych przekryć obiektów architektonicznych. Generowanie struktur poliedrycznych polega na łączeniu najbliższych wierzchołków "gałązek" konstrukcji fraktalnej a następnie wypełnieniu powstałej sieci płaszczyznami wieloboków.

Proces generowania konstrukcji fraktalnych jak również tworzenia przekryć oraz przykładowych wizualizacji architektonicznych zrealizowany został za pomocą programu komputerowego Autodesk VIZ 4.

Reviewer: Stanisław SULWIŃSKI, DSc, MSc

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