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SOME SPATIAL CONSTRUCTION OF COMMON POINTS OF TWO COAXIAL AND COPLANAR CONICS

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Abstract. Some planar problem of finding intersection points of two coaxial conics is solved by using a simple construction in the space. The construction is based on the theorem about the two quadrics intersection curve reducibility to two conics .

Key Words: conic, conic focus, conic axis, quadric, intersection curve of two quadrics, affinity, perspective collineation.

1. Introduction

The aim of this work is to find an intelligible way for students to construct the points of intersection of two coplanar conics with a common axis. In the case when one of the conics is a circle the construction presented by us is elementary as it is based on the well known theorem about the reducibility to two conics of the intersection curve of two quadrics which are circumscribed on the same sphere and therefore it is accessible to anyone with the knowledge of the basic descriptive geometry course. A general case may be too difficult for an average student because some experience in applying collineations is necessary.

Furthermore, the fact that a non-planar construction simplifies a planar problem can be appealing for certain students.

2. Construction of the Circle and Conic Intersection Points

We will construct the common points of a noncircular smooth conic and a circle whose centre lays on the conic axis. We give a detailed description of the construction in the case of a circle and an ellipse only.

The construction of the intersection points in the case of a circle and a parabola or a circle and a hyperbola is analogous.

Let be given: an ellipse s with axis AB i CD and a circle k, whose centre O lays on the line AB (Figure 1).

We consider two circular cones:

- a cone Φ , whose surface includes the ellipse s, is circumscribed on a sphere with an appropriate radius centred at O_1 and tangent to the plane of the ellipse s at the point K being a focus of s;
- a cone Ψ, with vertical axis and crossing the circle k , is circumscribed on a sphere inscribed in the cone Φ; the centre O_2 of the sphere is the common point of the axis l_1 of Φ and the axis l_2 of Ψ.

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It turns out that the intersection curve of the cones Φ and Ψ is obtained by two conic sections. At least one of them intersects the ellipse s and the circle k in their common points.

We execute the construction by using orthogonal projection onto a plane parallel to the plane η including both axis of the considered cones Φ and Ψ (Fig.1). Observe that the plane η is the symmetry plane of the intersection curve of Φ and Ψ.

We draw a circle tangent to the line x in the point K'. Next, we construct trough A' and B' the lines t_1' and t_2' tangent to this circle. These lines intersect at the vertex W_1'.

Now we inscribe in the angle $A'W_1'B'$ the circle with centre at $O_2' = l_1' \cap l_2'$, where the line l_1' is determined by O_1' and W_1', $O' \in l_2'$ and $l_2' \perp x$. The tangents t_3' and t_4' to this circle trough E' and F' respectively intersect the lines t_1' and t_2' at the following points:

$$I' = t_1' \cap t_3', \quad II' = t_2' \cap t_4', \quad III' = t_2' \cap t_3', \quad IV' = t_1' \cap t_4'.$$

The segments $I'\,II'$ and $III'\,IV'$ are the orthogonal projections of the conic sections which form the intersection curve of the cones Φ and Ψ circumscribed on the same sphere which is projected into the circle with centre at O_2'. The lines l_1 and l_2 (the axis of Φ and Ψ respectively) and the cones generators t_1, t_2 and t_3, t_4 are included in the symmetry plane η.

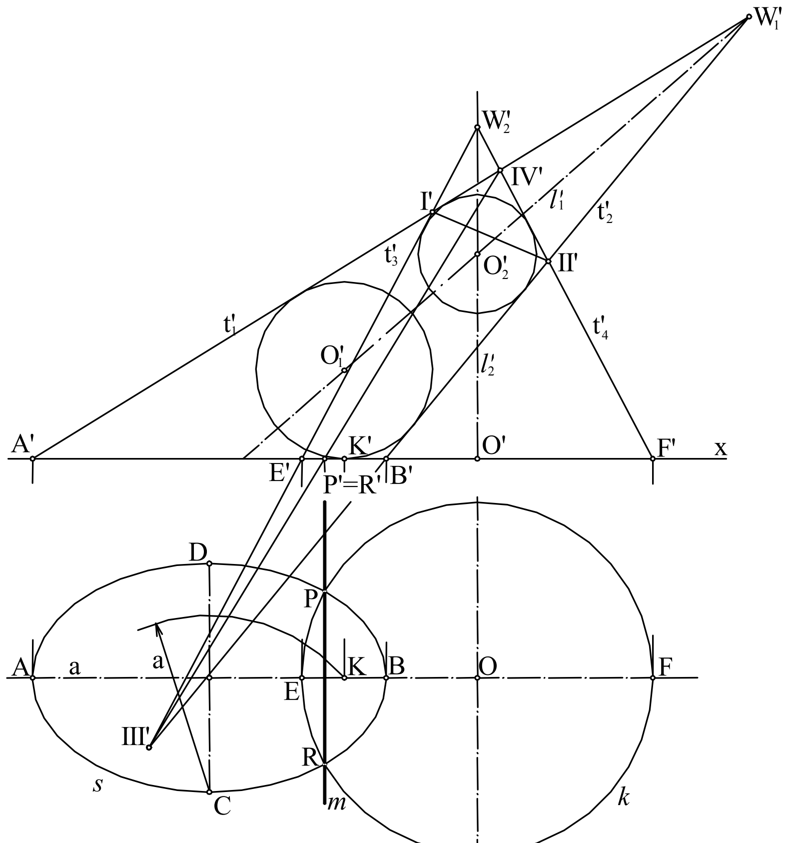

Figure 1

The line m which is the edge of the circle k plane and the plane of the conic defined by the projection $III'\,IV'$ intersects the circle k at the points P and R. Evidently the points P and R belong also to the ellipse s.

3. Corollary

The above procedure enables us to construct the common points of any two coaxial and coplanar conics. In this general case it suffices to transform one of the conics onto a circle in an appropriate way preserving the common axis of foci. It is possible if applying an infinity or a perspective collineation.

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PRZESTRZENNIE UZASADNIONA KONSTRUKCJA PUNKTÓW WSPÓLNYCH WSPÓŁOSIOWYCH STOŻKOWYCH

Rozwiązanie płaskiego zadania wyznaczania punktów wspólnych dwóch stożkowych położonych współosiowo uzyskano poprzez „wyjście w przestrzeń". Prosta konstrukcja oparta jest na znanym twierdzeniu o rozpadzie na dwie stożkowe linii przenikania dwóch powierzchni obrotowych opisanych na wspólnej kuli. Szczegółowy opis konstrukcji podano w przypadku elipsy i okręgu. Ogólny przypadek można sprowadzić do rozważanego przekształcając jedną ze stożkowych na okrąg przez stosowne powinowactwo lub kolineację środkową.

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